PME 3380 - Modelagem de Sistemas Dinâmicos

Exc 03/11

Gabriel Rodrigues Camargo - 10772460

① Pelos slides temos:

$$Y = \underbrace{(I + GH)^{-1} G}_{T} R \Rightarrow \quad T = (I + GH)^{-1} G$$

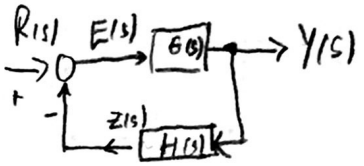



Diagrama de blocos de FTMF

$$Z = HY \Rightarrow \quad R - E = HY \Rightarrow R - G^{-1}Y = HY \Rightarrow R = (G^{-1} + H)Y \Rightarrow$$

$$\Rightarrow R = (I + HG)G^{-1}Y \Rightarrow \quad (I + HG)^{-1} R = G^{-1} Y \Rightarrow Y = G(I + HG)^{-1} R \Rightarrow$$

$$\boxed{\Rightarrow T = G(I + HG)^{-1}}$$

Logo prova-se

$$G(I + HG)^{-1} = (I + GH)^{-1} G = G(I + L)^{-1}$$

Onde usa-se $L = HG$

② $E = HY \Rightarrow Z - HGE = HG(R - Z) \Rightarrow HGR = (I + HG)Z \Rightarrow \boxed{(I + HG)^{-1} HG = ZR^{-1}}$

Também temos

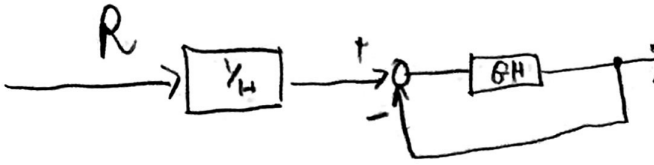
$$Y = GE \Rightarrow H^{-1}Z = G(R - Z) \Rightarrow GR = (H^{-1} + G)Z \Rightarrow GR = (I + GH)H^{-1}Z \Rightarrow$$

$$\Rightarrow \boxed{ZR^{-1} = H(I + GH)^{-1} G}$$

De mesma forma $L = HG$, logo

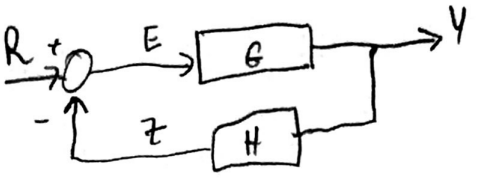
$$\frac{Z}{R} = \frac{L}{1+L} = \frac{GH}{1+GH} = \frac{HG}{1+HG} \rightarrow \quad Z = HY = HG(R - Z) \Rightarrow \quad \frac{Z}{R} = \frac{HG}{1+HG}$$

③

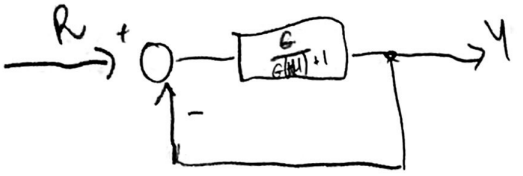



$Y = \left(\frac{R}{H} - Y\right) GH$ isolando $\Rightarrow Y = GR - YGH \Rightarrow$

$\Rightarrow \boxed{Y = \frac{GR}{1+GH}}$ → obtem-se em blocos



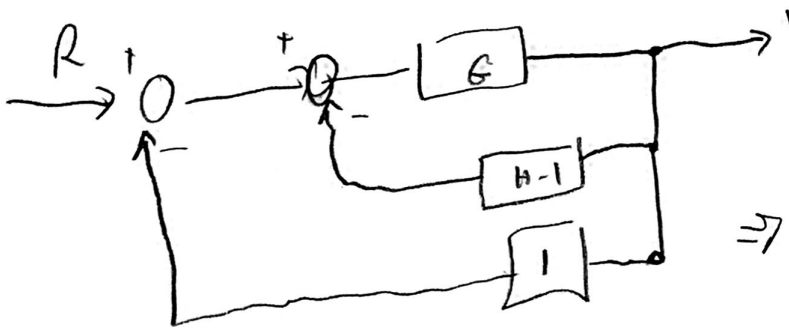

④




$Y = \left[\frac{G}{1+G(H-1)}\right](R-Y)$ isolando $\Rightarrow$

$\Rightarrow Y = \dfrac{\frac{GR}{1+G(H-1)}}{1+\frac{G}{1+G(H-1)}} \Rightarrow Y = \dfrac{GR}{G+1+G(H-1)} = \dfrac{GR}{1+GH} \Rightarrow \boxed{Y = \frac{GR}{1+GH}}$

⑤

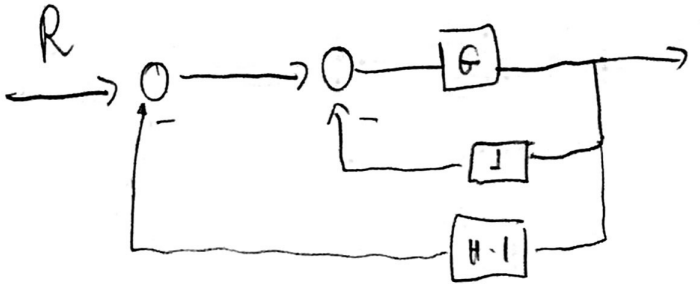



isolando

$\Rightarrow Y = [R - Y - Y(H-1)]\,G \Rightarrow$

$\Rightarrow \boxed{Y = \frac{GR}{1+GH}}$

⑥

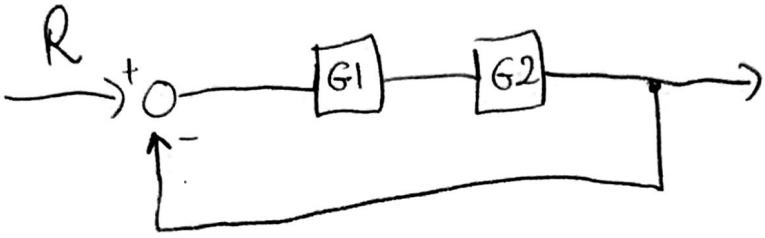



$\Rightarrow Y = [R - Y(H-1) - Y]\,G \Rightarrow$

$\Rightarrow Y + GY + GY(H-1) = R \Rightarrow$

$\Rightarrow \boxed{\frac{Y}{R} = \frac{G}{1+GH}}$

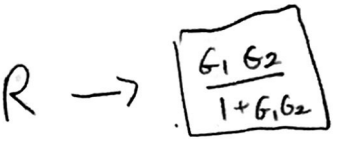
(7)




$$\Rightarrow Y_1 = (R - Y_1)\, G_1 G_2 \Rightarrow$$

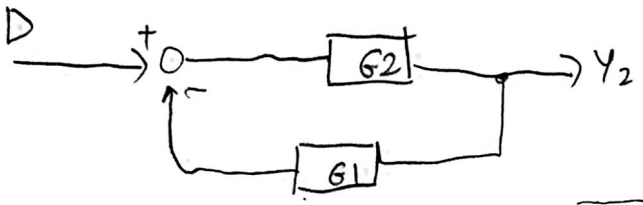
$$\Rightarrow Y_1 = \frac{R G_1 G_2}{1 + G_1 G_2}$$

em blocos:




(8)

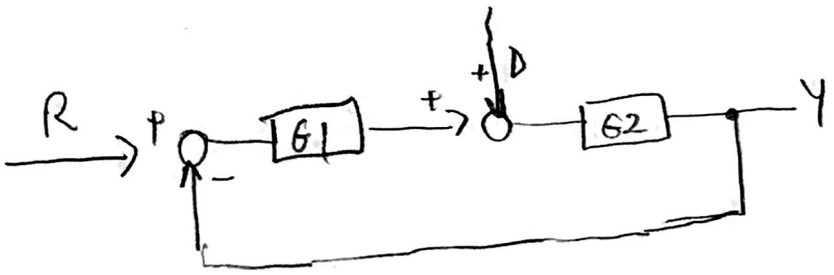
Com $R = 0$




$$\Rightarrow Y_2 = (D - G_1 Y_2) G_2 \Rightarrow$$

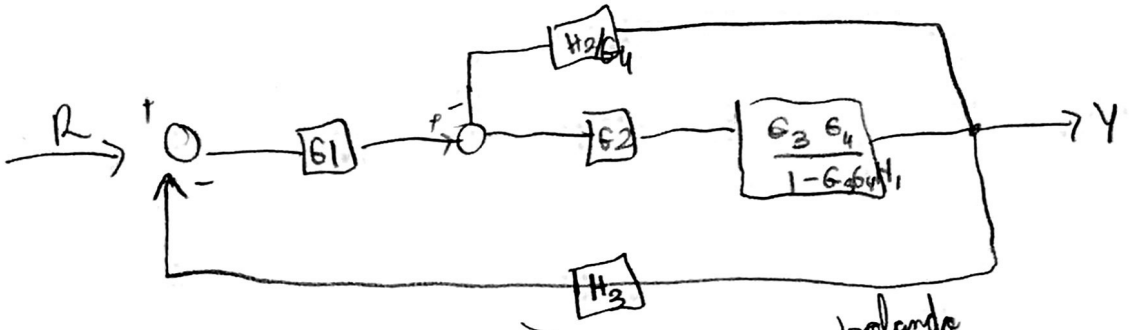
$$\Rightarrow Y_2 (1 + G_1 G_2) = D G_2 \Rightarrow \boxed{Y_2 = \frac{D G_2}{1 + G_1 G_2}}$$

(9)




$$Y = [(R - Y) G_1 + D]\, G_2 \Rightarrow Y(1 + G_1 G_2) = (R G_1 + D) G_2 \Rightarrow \boxed{Y = \frac{(R G_1 + D) G_2}{1 + G_1 G_2}}$$

(11)

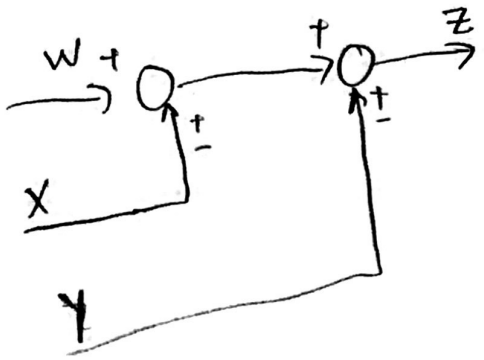



$$Y = \left[ (R - H_3 Y)\, G_1 - \frac{H_2}{G_4} Y \right] \frac{G_2 G_3}{1 - G_3 G_4 H_1}$$

balanda

$$\Rightarrow Y = \frac{G_1 G_2 G_3 G_4 R}{(1 - G_3 G_4 H_1 + G_1 G_2 G_3 G_4 H_3)}$$

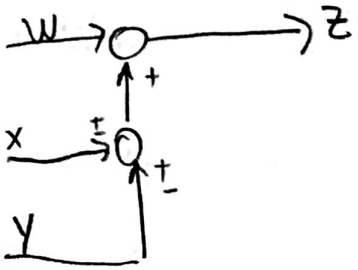
(10) Blocos iguais!




$\Rightarrow Z = (W \pm X) \pm Y = W \pm X \pm Y$

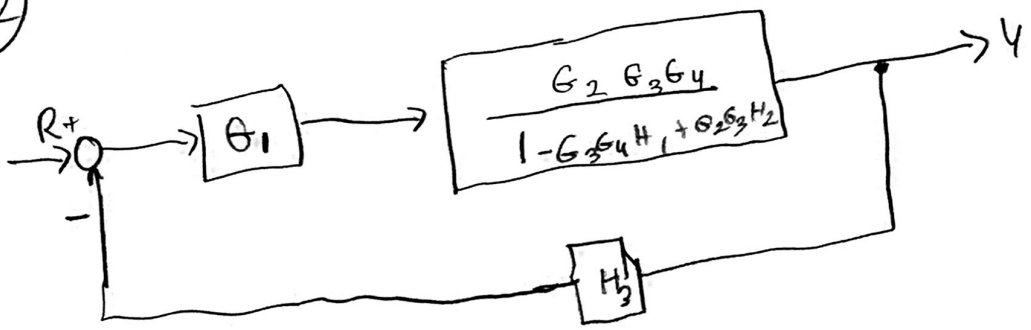
$\Updownarrow (=)$

$W \pm X \pm Y$




$\Rightarrow Z = (\pm X \pm Y) + W = W \pm X \pm Y$

(12)



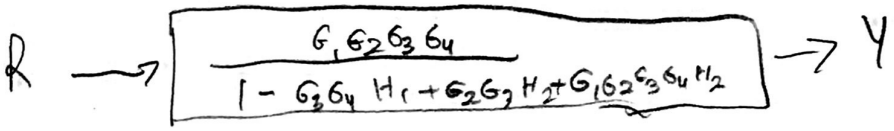

Logo

$$Y = \frac{G_2 G_3 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2} \left[ G_1 (R - H_3 Y) \right]$$

↓ isolando

$$Y = \frac{G_1 G_2 G_3 G_4 R}{1 - G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3}$$

(13)




Logo

$$Y = \frac{G_1 G_2 G_3 G_4 R}{1 - G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3}$$