PME 3380 - EXERCÍCIOS 22/10

Gabriela Vasconcelos Araujo - 10771497

1. $2\ddot{x} + 7\dot{x} + 3x = 0$; $x(0) = x_0$; $\dot{x}(0) = 0$

(transformada de Laplace)

$$2(s^2 X(s) - s\,x(0)) + 7(sX(s) - x_0) + 3X(s) = 0$$

$$(2s^2+7s+3)X(s) = (2s+7)x_0$$

$$X(s) = \frac{2s+7}{2s^2+7s+3}\,x_0 \qquad s_1 = (-7+5)/4 = -1/2 \qquad s_2 = (-7-5)/3 = -3$$

$$X(s) = \frac{(2s+7)x_0}{2(s+3)(s+1/2)} = \left[\frac{A}{(s+3)} + \frac{B}{(2s+1)}\right]\frac{x_0}{2} = \left[\frac{A(2s+1)+B(s+3)}{(s+3)(2s+1)}\right]\frac{x_0}{2}$$

$$\begin{cases} 2A + B = 2 \\ A + 3B = 7 \end{cases} \Rightarrow \quad A = -2/5, \quad B = 12/5$$

$$\therefore\ X(s) = \frac{-x_0}{5(s+3)} + \frac{6x_0}{5(s+1/2)}$$

$$\mathcal{L}^{-1}\{1/(s-a)\} = e^{at}$$

$$x(t) = -\frac{x_0}{5}e^{-3t} + \frac{6}{5}x_0 e^{-t/2}$$

2. $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$; $\ddot{x}(0) = 2$; $\dot{x}(0) = 1$; $x(0) = 9$; $u(0) = 1$; $\dot{u}(0) = 0$

(transformada de Laplace)

$$s^3X(s) - s^2x(0) - s\dot{x}(0) - \ddot{x}(0) + 2(s^2X(s) - s\,x(0) - \dot{x}(0)) + 7(sX(s) - x(0))$$

$$= s^2U(s) - sU(0) - \dot{U}(0) + 7(sU(s) - u(0)) + 5U(s)$$

$$\therefore\ X(s)(s^3+2s^2+7s) = U(s)(s^2+7s+5) + 9s^2+18s+60$$

$$X(s) = \frac{U(s)(s^2+7s+5)}{(s^3+2s^2+7s)} + \frac{9s^2+18s+60}{(s^3+2s^2+7s)}$$

$$G(s) = \frac{X(s)}{U(s)} \Rightarrow G(s) = \frac{s^2+7s+5}{s(s^2+2s+7)}$$

polos: $s_1 = 0$, $s_2 = -1 + \sqrt{5}\,i$, $s_3 = -1 - \sqrt{5}\,i$

$\therefore$ sistema é estável

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Pelo método de frações parciais:

$$X(s) = \frac{9s^3 + 19s^2 + 67s + 5}{s^2(s^2+2s+7)} \Rightarrow X(s) = \frac{a}{s} + \frac{b}{s^2} + \frac{cs+d}{s^2+2s+7}$$

$$a = \frac{459}{49} \; ; \; b = \frac{5}{7} \; ; \; c = -\frac{18}{49} \; ; \; d = \frac{22}{49}$$

$$\therefore X(s) = \frac{1}{49}\left[\frac{459}{s} + \frac{245}{7s^2} - \frac{18s+22}{s^2+2s+7}\right]$$

$$X(s) = \frac{1}{49}\left[\frac{459}{s} + \frac{245}{7s^2} - \frac{18(s+1)}{(s+1)^2+6} - \frac{4}{(s+1)^2+6}\right]$$

$$\mathcal{L}^{-1}[X(s)] = x(t)$$

$$x(t) = \frac{459}{9} + \frac{5t}{7} - \frac{18}{49}e^{-t}\cos(\sqrt{6}t) - \frac{4}{49\sqrt{6}}e^{-t}\,\text{sen}(\sqrt{6}t)$$

spiral