

1.

$$C_1 = \frac{1}{1!} \left[\frac{\partial}{\partial s} \frac{1}{(s+1)^2 (s+2)} \right]_{s=-1} = \frac{-1}{(s+2)^2} = -1$$

$$C_2 = \frac{1}{0!} \left[\frac{1}{(s+1)^2 (s+2)} \right]_{s=-1} = \frac{1}{1} = 1$$

$$C_3 = \frac{1}{0!} \left[\frac{1}{(s+1) (s+2)} \right]_{s=-2} = 1$$

$$G(s) = \frac{-1}{(s+1)} + \frac{1}{(s+1)^2} + \frac{1}{(s+2)}$$

$$g(t) = -e^{-t} - te^{-t} + e^{-2t}$$

2. $2\ddot{x} + 7\dot{x} + 3x = 0$

$$2 \cdot \mathcal{L}(\ddot{x}) = 2 \cdot s^2 X(s) - sX(0) - \dot{x}(0)$$

$$7 \cdot \mathcal{L}(\dot{x}) = 7 \cdot s X(s) - X(0)$$

$$3 \cdot \mathcal{L}(x) = 3 \cdot X(s)$$

$$(2s^2 + 7s + 3) X(s) = (s+1) X_0$$

$$X(s) = \frac{(s+1)}{(2s^2 + 7s + 3)} X_0$$

$$\frac{s+1}{2(s+3)(s+0,5)} \Rightarrow \frac{1}{2} \left(\frac{C_1}{s+3} + \frac{C_2}{s+0,5} \right) \Rightarrow C_1 = \frac{s+1}{(s+0,5)} \Big|_{s=-3} = \frac{-2}{-2,5} = 0,8$$

$$X(s) = \frac{X_0}{2} \left(\frac{0,8}{s+3} + \frac{0,2}{s+0,5} \right)$$

$$C_2 = \frac{s+1}{s+3} \Big|_{s=-0,5} = \frac{0,5}{2,5} = 0,2$$

$$x(t) = \frac{X_0}{2} (0,8 \cdot e^{-3t} + 0,2 \cdot e^{-0,5t})$$

3. $\ddot{x} + 2\dot{x} + 7x = \ddot{u} + 7\dot{u} + 5u$

$$\mathcal{L}(\ddot{x}) = s^2 X(s) - s^2 \cdot 9 - s \cdot 1 - 2$$

$$2 \cdot \mathcal{L}(\ddot{x}) = 2(s^2 X(s) - 9s - 1)$$

$$7 \cdot \mathcal{L}(\dot{x}) = 7(s X(s) - 9)$$

$$\mathcal{L}(\ddot{u}) = s^2 U - s \cdot 9 - 0 = 0$$

$$7 \cdot \mathcal{L}(\dot{u}) = 7(s U - 1) = 0$$

$$5 \mathcal{L}(u) = 5 U = 5/s$$

$$X(s) (s^3 + 2s^2 + 7s) - s^2 \cdot 9 - 19s - 67 = 5/s$$

$$X(s) = \frac{5}{s[(s^3 + 2s^2 + 7s) - s^2 \cdot 9 - 19s - 67]} = \frac{5}{s[s^3 - 7s^2 - 12s - 67]}$$

$$X(s) = \frac{5}{s(s-9,121)(s^2 + 2,121s + 7,346)}$$

$$X(s) = \frac{C_1}{s} + \frac{C_2}{(s-9,121)} + \frac{C_3 s + C_4}{(s^2 + 2,121s + 7,346)}$$

$$C_1 = \frac{5}{(s-9,121)(s^2 + 2,121s + 7,346)} \Big|_{s=0} = \frac{5}{-9,121 \cdot 7,346} = -0,075$$

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Partial Fraction Decomposition

$$C_2 = \frac{5}{s(s^2 + 2,121s + 7,346)} \Big|_{s=9,121} = \frac{5}{9,121(9,121^2 + 2,121 \cdot 9,121 + 7,346)} = 0,005$$

$$= 0,07s \cdot (s-9,121)(s^2 + 2,121s + 7,346) + 0,005(s^2 + 2,121s + 7,346)s + (C_3s + C_4)s(s-9,121) = 5$$

$$C_3 = 0,07$$

$$C_4 = 2$$