$\mathcal{L}^{-1}\left\{\frac{1}{s-\alpha}\right\} = e^{\alpha t}$

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1) $2\ddot{x} + 7\dot{x} + 3x = 0$, com $x(0) = x_0$ e $\dot{x}(0) = 0$

- Transformada de Laplace: $2\mathcal{L}\{\ddot{x}\} + 7\mathcal{L}\{\dot{x}\} + 3\mathcal{L}\{x\} = 0$

$$\to 2\left(s^2X(s) - sX(0) - \overset{=0}{\dot{x}(0)}\right) + 7\left(sX(s) - x(0)\right) + 3X(s) = 0$$

$$\therefore (2s^2 + 7s + 3)X(s) = (2s+7)x_0 \to \boxed{X(s) = \frac{x_0(2s+7)}{2(s+3)(s+1/2)}}$$

- Podemos escrever que: $X(s) = \frac{x_0(2s+7)}{(s+3)(2s+1)} = \frac{A}{s+3} + \frac{B}{2s+1} = \frac{s(2A+B)+(A+3B)}{(s+3)(2s+1)}$

$$\begin{cases} 2A + B = 2x_0 & \to B = \frac{12}{5}x_0 \\ A + 3B = 7x_0 & \to A = -\frac{1}{5}x_0 \end{cases} \to \boxed{X(s) = -\frac{x_0}{5(s+3)} + \frac{12x_0}{5(2s+1)}}$$

- Transformada inversa: $x(t) = -\frac{x_0}{5}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} + \frac{6x_0}{5}\mathcal{L}^{-1}\left\{\frac{1}{s+1/2}\right\}$

$$\therefore \boxed{x(t) = -\frac{x_0}{5}e^{-3t} + \frac{6x_0}{5}e^{-t/2}}$$

2) $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$; $\ddot{x}(0) = 2$ / $\dot{x}(0) = 1$ / $x(0) = 9$ / $\dot{u}(0) = 0$, $u(0) = 0$

- Transformada de Laplace: $\left[s^3X(s) - \ddot{x}(0) - s\dot{x}(0) - s^2x(0)\right] + 2\left[s^2X(s) - \dot{x}(0) - sX(0)\right] +$
$7\left[sX(s) - X(0)\right] = \left[s^2U(s) - \overset{0}{\dot{u}(0)} - s\overset{0}{u(0)}\right] + 7\left[sU(s) - \overset{0}{u(0)}\right] + 5U(s)$

$$\therefore s^3X(s) - 2 - s - 9s^2 + 2s^2X(s) - 2 - 18s + 7sX(s) - 63 = s^2U(s) + 7sU(s) + 5U(s)$$

$$(s^3 + 2s^2 + 7s)X(s) = 9s^2 + 19s + 67 + (s^2 + 7s + 5)U(s) \to U(s) = \frac{1}{s}$$

$$\therefore X(s) = \frac{9s^3 + 20s^2 + 74s + 5}{s^2(s^2 + 2s + 7)} = \boxed{\frac{508}{49s} + \frac{5}{7s^2} - \frac{(67s + 71)}{49[(s+1)^2 + 6]}}$$

$$- \frac{4}{49}\,\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+6}\right\}$$

Transformada inversa:

$$x(t) = \frac{508}{49}\cdot\overbrace{\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}}^{1} + \frac{5}{7}\cdot\overbrace{\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}}^{t} - \frac{67}{49}\cdot\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+6}\right\}$$

$$\therefore \boxed{x(t) = \frac{508}{49} + \frac{5t}{7} - \frac{67}{49}\cdot e^{-t}\cdot\cos(t\sqrt{6}) - \frac{4}{49\sqrt{6}}\cdot e^{-t}\cdot \operatorname{sen}(t\sqrt{6})}$$