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1) $2\ddot{x} + 7\dot{x} + 3x = 0$, $\begin{cases} x(0) = x_0 \\ \dot{x}(0) = 0 \end{cases}$

$\Rightarrow$

$2\left(s^2 X(s) - s\,x(0) - \cancel{\dot{x}(0)}\right) + 7\left(s X(s) - x(0)\right) + 3X(s) = 0$

$$X(s)\left(2s^2 + 7s + 3\right) - 2s\,x(0) - 7x(0) = 0$$

$$\Rightarrow X(s) = \frac{2s x_0 + 7x_0}{2s^2 + 7s + 3} = \frac{x_0(2s+7)}{2(s+3)(s+\frac{1}{2})}$$

$\Rightarrow X(s) = \frac{A}{s+3} + \frac{B}{2s+1} \Rightarrow 2As + A + Bs + 3B = 2s x_0 + 7x_0$

$$\Rightarrow \begin{cases} 2A + B = 2x_0 \\ A + 3B = 7x_0 \end{cases}$$

$-5B = -12x_0 \Rightarrow$

$B = \frac{12}{5} x_0$

$\Rightarrow A = \frac{-x_0}{5}$

$\Rightarrow X(s) = \frac{-x_0}{5(s+3)} + \frac{12x_0}{5(2s+1)}$ $\quad \mathcal{L}^{-1}\left(\frac{1}{s-a}\right) = e^{a t}$

$\Rightarrow \cancel{X(s)}$ $\boxed{x(t) = \frac{-x_0}{5}\left(e^{-3t}\right) + \frac{12x_0}{10}\left(e^{-\frac{1}{2}t}\right)}$

2) $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$ $\quad \begin{cases} \ddot{x}(0) = 2 \\ \dot{x}(0) = 1 \\ x(0) = 9 \end{cases}$ $\begin{cases} \dot{u}(0) = 0 \\ u(0) = 0 \end{cases}$

$\Rightarrow$

$$s^3X(s) - s^2x(0) - s\dot{x}(0) - \ddot{x}(0) + 2\left(s^2X(s) - sx(0) - \dot{x}(0)\right) +$$

$$+ 7\left(sX(s) - x(0)\right) = s^2U(s) - s\cancel{u(0)}^{0} - \cancel{\dot{u}(0)}^{0} + 7\left(sU(s) - \cancel{u(0)}^{0}\right) + 5U(s)$$

$$\Rightarrow X(s)\left[s^3 + 2s^2 + 7s\right] = U(s)\left[s^2 + 7s + 5\right] + \underbrace{9s^2 + s + 2 + 18s + 2 + 63}_{9s^2 + 19s + 67}$$

$$\Rightarrow X(s) = U(s)\left(\frac{s^2 + 7s + 5}{s^3 + 2s^2 + 7s}\right) + \left(\frac{9s^2 + 19s + 67}{s^3 + 2s^2 + 7s}\right)$$

$$\Rightarrow u(t) = 1 \Rightarrow U(s) = \frac{1}{s}$$

$$\Rightarrow X(s) = \frac{s^2 + 7s + 5 + s(9s^2 + 19s + 67)}{s^2(s^2 + 2s + 7)} = \frac{9s^3 + 20s^2 + 74s + 5}{s^2(s^2 + 2s + 7)}$$

$$\Rightarrow X(s) = \frac{508}{49s} + \frac{5}{7s^2} + \frac{-67s - 71}{49\left((s+1)^2 + 6\right)}$$

$$\Rightarrow x(t) = \frac{508}{49} + \frac{5}{7}t - \frac{67}{49}e^{-t}\cos(\sqrt{6}t) - \frac{4}{49\sqrt{6}}e^{-t}\sin(\sqrt{6}t)$$