PME 3380 - Exc. 22.10

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① $2\ddot{x} + 7\dot{x} + 3x = 0$; $x(0) = x_0$; $\dot{x}(0) = 0$

Aplicando tranformada $\mathcal{L}$:

$$2s^2X - 2x_0 s + 7sX - 7x_0 + 3X = 0 \Rightarrow (-2-7)\cancel{x_0} = (2s^2+7s+3)$$

$$\Rightarrow X(s) = \frac{2s+7}{2s^2+7s+3}x_0 \Rightarrow X(s) = \left(\frac{a}{s+3} + \frac{b}{s+\frac{1}{2}}\right)\frac{x_0}{2} \Rightarrow$$

$$\Rightarrow X(s) = \left[\frac{-2}{s(s+3)} + \frac{12}{s(s+\frac{1}{2})}\right]\frac{x_0}{2} \Rightarrow X(s) = \frac{6x_0}{s(s+\frac{1}{2})} - \frac{x_0}{s(s+3)} \overset{\mathcal{L}^{-1}}{\Rightarrow}$$

$$\Rightarrow x(t) = -\frac{x_0}{5}e^{-3t} + \frac{6x_0}{5}e^{-t/2}$$

② $\dddot{X} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$; $u(t) = 1$; $u(0) = 1$, $\dot{u}(0) = 0$; $x(0) = 9$, $\dot{x}(0) = 1$, $\ddot{x}(0) = 2$

$\downarrow \mathcal{L}$: $U(s) = \frac{1}{s}$

$\downarrow \mathcal{L}$

$$s^3X - s^2x(0) - s\dot{x}(0) - \ddot{x}(0) + 2(s^2X - sx(0) - \dot{x}(0)) + 7(sX - x(0)) =$$
$$s^2U - su(0) - \dot{u}(0) + 7(sU - u(0)) + 5U \Rightarrow$$

$$\Rightarrow (s^3 + 2s^2 + 7s)X = (s^2 + 7s + 5)U + s^2x(0) + s(\dot{x}(0) + 2x(0) - u(0)) +$$
$$+ (\ddot{x}(0) + 2\dot{x}(0) + 7x(0) - \dot{u}(0) - 7u(0)) \Rightarrow$$

$$\Rightarrow (s^3 + 2s^2 + 7s)X = (s^2 + 7s + 5)U + 9s^2 + 18s + 60 \Rightarrow$$

$$\Rightarrow X(s) = \frac{s^2 + 7s + 5}{s^2(s^2 + 2s + 7)} + \frac{9s^2 + 18s + 60}{s(s^2 + 2s + 7)}$$

↳ Sol particular ↳ Sol homogênea

$$G(s) = \frac{X(s)}{U(s)} \Rightarrow G(s) = \frac{s^2+7s+5}{s(s^2+2s+7)}$$

$s=0$; $s=-1+2{,}449j$; $s=-1-2{,}449j$

↳ Sist estável

Escrevemos

$$X(s) = \frac{a}{s} + \frac{b}{s^2} + \frac{cs+d}{s^2+2s+7}$$

Fazendo um desenvolvimento algebrico chegamos:

$$X(s) = \frac{1}{49}\left[\frac{459}{s} + \frac{245}{7s^2} - 18\frac{(s+1)}{(s+1)^2+6} - \frac{4}{(s+1)^2+6}\right]$$

$\downarrow \mathcal{L}^{-1}$

$$\boxed{x(t) = \frac{459}{9} + \frac{5}{7}t - \frac{18}{49}e^{-t}\cos(\sqrt{6}t) - \frac{4}{49\sqrt{6}}e^{-t}\sin(\sqrt{6}t)}$$