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$$G(s) = \frac{1}{(s+1)^2 (s+2)} = \frac{C_1}{(s+1)} + \frac{C_2}{(s+1)^2} + \frac{C_3}{(s+2)}$$

- $C_3 = G(s)\cdot(s+2)\Big|_{s=-2} = \frac{1}{(s+1)^2}\Big|_{s=-2} = \frac{1}{-1^2} = 1$

- $C_{k-i} = \frac{1}{i!}\left[\frac{d^i}{ds^i}(s+p_i)^k \cdot G(s)\right]\Bigg|_{s=-p_1}$

- Para $i = 0$, $k = 2$ $(p_1 = +1)$:

  - $C_{2-0} = \frac{1}{1}\left[\frac{d^0}{ds^0}(s+p_1)^2 \cdot G(s)\right]\Bigg|_{s=-1} =$

$$= (s+1)^2 \cdot \frac{1}{(s+1)^2(-1+2)} = 1$$

- Para $i = 1$, $(k=2)$

  - $C_{2-1} = \frac{1}{1}\left[\frac{d}{ds}(s+p)^2 \cdot G(s)\right]\Bigg|_{s=-1} =$

$$= \frac{d}{ds}\left(\frac{1}{s+2}\right)\Bigg|_{s=-1} = \frac{-1}{(s+2)^2}\Bigg|_{s=-1} = -1$$

$$G(s) = \frac{-1}{(s+1)} + \frac{1}{(s+1)^2} + \frac{1}{(s+2)}$$

1) Achar as FTs e resolver as EDOs

- $2\ddot{x} + 7\dot{x} + 3x = 0$; $x(0) = x_0$; $\dot{x}(0) = 0$

- FT: $\downarrow \mathcal{L}$, CI nulas

$$2s^2 X(s) + 7\cdot s X(s) + 3X(s) = 1 \quad \text{degrau}$$

$$\frac{X(s)}{U(s)} = \frac{1}{2s^2 + 7s + 3}$$

- Resolver a EDO:

  - $\mathcal{L}(f(t))$: $2(s^2 X(s) - sX(0) - \dot{x}(0)) + 7(sX(s) - x(0)) + 3X(s) = 0$

$$\Rightarrow 2s^2 X(s) - 2\cdot s x_0 + 7sX(s) - 7x_0 + 3X(s) = 0 \Rightarrow$$

$$\Rightarrow X(s)\cdot(2s^2 + 7s + 3) = (2s + 7)x_0 \Rightarrow$$

$$\Rightarrow X(s) = \frac{(2s+7)}{(2s^2+7s+3)} x_0$$

· Separando em frações parciais:

Polos: -3 e $-\frac{1}{2}$

$$X(s) = \left(\frac{\alpha_1}{(s+3)} + \frac{\alpha_2}{(s+\frac{1}{2})}\right)\frac{x_0}{2}$$

$$\cdot\ \alpha_1 = (s+3)\cdot X(s)\Big|_{s=-3} = \frac{2\cdot(-3)+7}{(-3+\frac{1}{2})} = -0{,}4$$

$$\cdot\ \alpha_2 = (s+\tfrac{1}{2})\cdot X(s)\Big|_{s=-\frac{1}{2}} = \frac{2(-\frac{1}{2})+7}{(-\frac{1}{2}+3)} = 2{,}4$$

$$X(s) = -\frac{0{,}4}{(s+3)}\cdot\frac{1}{2}x_0 + \frac{2{,}4}{(s+\frac{1}{2})}\frac{1}{2}x_0$$

$$X(s) = -\frac{0{,}2\,x_0}{(s+3)} + \frac{1{,}2\,x_0}{(s+\frac{1}{2})}$$

$\downarrow \mathcal{L}^{-1}$

$$x(t) = -0{,}2\cdot x_0\cdot e^{-3t} + 1{,}2\cdot x_0\cdot e^{-\frac{1}{2}t}$$

2) · $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u$; $\ddot{x}(0) = 2$; $\dot{x}(0) = 1$; $x(0) = 9$; $\dot{u}(0) = 0$; $u(0) = 1$

• FT: $\mathcal{L}$, CI nula:

$$\Rightarrow s^3X(s) + 2s^2X + 7sX = s^2U + 7sU + 5U \Rightarrow$$

$$\Rightarrow X(s)\cdot(s^3+2s^2+7s) = U(s)\cdot(s^2+7s+5) \Rightarrow$$

$$\Rightarrow \frac{X(s)}{U(s)} = \frac{s^2+7s+5}{s\cdot(s^2+2s+7s)}$$

• Resolver a EDO:

$$\cdot\ \mathcal{L}(f(t)):\ s^3X(s) - s^2x(0) - s\dot{x}(0) - \ddot{x}(0) +$$
$$+ 2(s^2X(s) - sx(0) - \dot{x}(0)) + 7(sX(s) - x(0))$$
$$= U(s)\cdot s^2 - su(0) - \dot{u}(0) + 7(sU(s) - u(0))$$
$$+ 5\cdot U(s) \Rightarrow$$

$$\Rightarrow s^3X(s) - s^2\cdot 9 - s\cdot 1 - 2 + 2s^2X(s) - 2\cdot s\cdot 9 - 2 + 7sX(s) - 7\cdot 9 =$$
$$= U(s)\cdot s^2 - s + 7sU(s) - 7 + 5\cdot U(s) \Rightarrow$$

$$\Rightarrow X(s)\cdot(s^3+2s^2+7s) - (9s^2+19s+67) =$$
$$= U(s)\cdot(s^2+7s+5) - (s+7)$$

$$X(s) = \frac{U(s)\cdot(s^2+7s+5)}{s^3+2s^2+7s} + \frac{9s^2+19s+67}{s^3+2s^2+7s} - \frac{s+7}{s^3+2s^2+7s}$$

Considerando $u(t) = 1$; $U(s) = \frac{1}{s}$:

$$X(s) = \frac{(s^2+7s+5)}{s^4+2s^3+7s^2} + \frac{9s^2+19s+67}{s^3+2s^2+7s} - \frac{s+7}{s^3+2s^2+7s}$$

$$X_1(s) = \frac{s^2+7s+5}{s^2(s^2+2s^3+7s^2)} + \frac{9s^2+18s+60}{s^3+2s^2+7s}$$

• Expandido em frações parciais:

$$X(s) = \frac{\frac{5}{7}}{s^2} + \frac{\frac{39}{49}}{s} - \frac{\left(\frac{39}{49}s+\frac{64}{49}\right)}{s^2+2s+7} + \frac{\frac{60}{7}}{s} + \frac{\frac{3}{7}s+\frac{6}{7}}{s^2+2s+7} =$$

$$= \frac{\frac{5}{7}}{s^2} + \frac{\frac{39}{49}}{s} - \frac{39s+64}{49((s+1)^2+6)} + \frac{\frac{60}{7}}{s} + \frac{3s+6}{7\cdot((s+1)^2+6)}$$

$$= \frac{\frac{5}{7}}{s^2} + \frac{\frac{459}{49}}{s} - \frac{18(s+1)}{49((s+1)^2+6)}$$

• No domínio do tempo:

$$x(t) = \frac{5}{7}\cdot t + \frac{459}{49} - \frac{18}{49}e^{-t}\cdot\cos(\sqrt{6}\cdot t)$$