Aula 22/10

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1) $2\ddot{x} + 7\dot{x} + 3x = 0 \qquad x(0) = x_0 \quad \dot{x}(0) = 0$

$$\begin{cases} 2\ddot{x} \Rightarrow 2\cdot[s^2X(s) - sx(0) - \dot{x}(0)] = 2[s^2X(s) - sx_0] \\ 7\dot{x} \Rightarrow 7[sX(s) - x(0)] = 7[sX(s) - x_0] \\ 3x \Rightarrow 3X(s) \end{cases}$$

$$(2s^2 + 7s + 3)X(s) - (2s + 7)x_0 = 0$$

$$X(s) = \frac{(2s+7)x_0}{(2s^2+7s+3)}, \quad G(s) = \frac{X(s)}{U(s)} \qquad G(s) = 0 \rightarrow \text{não há entrada}$$

↳ Cálculo dos Pólos: $2s^2 + 7s + 3 = 0 \rightarrow s = -0,5 \quad s = -3$

↳ Cálculo da Transformada Inversa

$$X(s) = \frac{(2s+7)x_0}{2(s+0,5)(s+3)} = \left(\frac{\alpha_1}{s+0,5} + \frac{\alpha_2}{s+3}\right)\frac{x_0}{2}$$

$$\begin{cases} \alpha_1 = \left.\dfrac{2s+7}{s+3}\right|_{s=-0,5} = 2,4 \\ \alpha_2 = \left.\dfrac{2s+7}{s+0,5}\right|_{s=-3} = -0,4 \end{cases}$$

$$X(s) = \left(\frac{2,4}{s+0,5} - \frac{0,4}{s+3}\right)\frac{x_0}{2} = \left(\frac{1,2}{s+0,5} - \frac{0,2}{s+3}\right)x_0$$

Feminina

↳ Transformada inversa

$$X(t) = 1{,}2x_0 \cdot e^{-0{,}5t} - 0{,}2e^{-3t}$$

② $\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u \qquad \ddot{x}(0) = 2;\ \dot{x}(0) = 1;\ x(0) = 9$

$u(0) = 1;\ \dot{u} = 0;\ u(t) = 1$

$\dddot{x} = s^3X(s) - s^2X(0) - s\dot{x}(0) - \ddot{x}(0)$

$= s^3X(s) - 9s^2 - s - 2$

$2\ddot{x} = (s^2X(s) - sx(0) - \dot{x}(0))2 = 2(s^2X(s) - 9s - 1)$

$7\dot{x} = (sX(s) - x(0))7 = 7(sX(s) - 9)$

$\ddot{u} = (s^2U(s) - su(0) - \dot{u}(0)) = s^2U(s) - s$

$7\dot{u} = 7(sU(s) - u(0)) = 7(sU(s) - 1)$

$5u = 5U(s)$

$(s^3 + 2s^2 + 7s)X(s) - 9s^2 - 19s - 67 = (s^2 + 7s + 5)U(s) - s - 7$

$(s^3 + 2s^2 + 7s)X(s) = (s^2 + 7s + 5)U(s) + 9s^2 + 18s + 60$

$$X(s) = \frac{(s^2 + 7s + 5)U(s)}{(s^3 + 2s^2 + 7s)} + \frac{(9s^2 + 18s + 60)}{(s^3 + 2s^2 + 7s)}$$

$$G(s) = \frac{s^2 + 7s + 5}{s^3 + 2s^2 + 7s}$$

↳ Pólos: $(s^3+2s^2+7s) = s(s^2+2s+7) \Rightarrow s=0$

$s = -1-2\sqrt{6}$

$s = -1+2\sqrt{6}$

sendo $u(t)=1 \Rightarrow U(s) = \frac{1}{s}$

$$X(s) = \frac{s^2+7s+5+9s^3+18s^2+60s}{s^2(s^2+2s+7)} - \frac{9s^3+19s^2+67s+5}{s^2(s^2+2s+7)}$$

$$X(s) = \frac{\alpha_1}{s} + \frac{\alpha_2}{s^2} + \frac{\alpha_3 \cdot s + \alpha_4}{s^2+2s+7}$$

$$\alpha_2 = X(s)\cdot s^2 \big|_{s=0} = 5/7 \;//$$

$$\alpha_1 = \frac{d}{ds}\left(X(s)\cdot s^2\right)\Big|_{s=0} = \frac{(27s^2+38s+67)(s^2+2s+7) - (9s^3+19s^2+67s+5)(2s+2)}{(s^2+2s+7)^2}\Bigg|_{s=0}$$

$$= \frac{469}{49} - \frac{10}{49} = \frac{459}{49} \;//$$

∴ Para α_3 e α_4 (Igualando os denominadores)

$$\frac{459}{49}\cdot s(s^2+2s+7) + \frac{5}{7}(s^2+2s+7) + s^2(\alpha_3 s + \alpha_4) = 9s^3+19s^2+67s+5$$

$$459\,s(s^2+2s+7) + 35(s^2+2s+7) + 49s^2(\alpha_3 s+\alpha_4) = 49(9s^3+19s^2+67s+5)$$

$$(459+49\alpha_3)s^3 + (918+35+49\alpha_4)s^2 = 49(9s^3+19s^2)$$

$$\begin{cases} 459+49\alpha_3 = 49\cdot 9 \Rightarrow \alpha_3 = -18/49 \\ 918+35+49\alpha_4 = 49\cdot 19 \Rightarrow \alpha_4 = -22/49 \end{cases}$$

$$X(s) = \frac{459}{49}\cdot\frac{1}{s} + \frac{5}{7}\cdot\frac{1}{s^2} - \frac{18s/49 + 22/49}{s^2+2s+7}$$

$$X(s) = \frac{1}{49}\left[459\,\frac{1}{s} + 35\,\frac{1}{s^2} - \frac{18s+22}{s^2+2s+7}\right]$$

$$X(s) = \frac{1}{49}\left[459\,\frac{1}{s} + 35\,\frac{1}{s^2} - \frac{18(s+1)}{(s+1)^2+6} - \frac{4}{(s+1)^2+6}\right]$$

$$x(t) = \frac{1}{49}\left[459 + 35t - 18\,e^{-t}\cos\sqrt{6}t - \frac{4}{\sqrt{6}}\,e^{-t}\operatorname{sen}\sqrt{6}t\right]$$