Modelagem

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→ Resolver as EDO's

1)

$$2\ddot{X} + 7\dot{X} + 3X = 0 \quad ; \quad X(0) = X_0 \ \text{e} \ \dot{X}(0) = 0$$

- Aplicando a transformada de laplace:

$$2\mathcal{L}\{\ddot{X}\} + 7\mathcal{L}\{\dot{X}\} + 3\mathcal{L}\{X\} = 0$$

$$2(s^2 X(s) - sX(0) - \cancel{\dot{X}(0)}^{0}) + 7(sX(s) - X(0)) + 3X(s) = 0$$

$$2s^2 X(s) + 7sX(s) + 3X(s) = 2sX_0 + 7X_0$$

$$X(s) = \frac{X_0(2s+7)}{2s^2+7s+3} = \frac{X_0(2s+7)}{2(s+3)(s+1/2)}$$

- Usando o método das frações parciais:

$$\frac{X_0(2s+7)}{(s+3)(2s+1)} = \frac{A}{(s+3)} + \frac{B}{(2s+1)} = \frac{2sA + A + Bs + 3B}{(s+3)(2s+1)}$$

$$\left| \begin{array}{l} 2A + B = 2X_0 \\ A + 3B = 7X_0 \end{array} \right. \rightarrow \begin{array}{l} A = -\frac{1}{5}X_0 \\ B = \frac{12}{5}X_0 \end{array}$$

$$\therefore X(s) = -\frac{X_0}{5(s+3)} + \frac{12X_0}{5(2s+1)}$$

Aplicando a Transformada inversa: $\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{a\tau}$

$$X(\tau) = -\frac{X_0}{5}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} + \frac{6}{5}X_0\mathcal{L}^{-1}\left\{\frac{1}{s+1/2}\right\} \rightarrow \boxed{X(\tau) = -\frac{X_0}{5}.e^{-3\tau} + \frac{6X_0}{5}.e^{-\frac{1}{2}\tau}}$$

2)

$$\dddot{x} + 2\ddot{x} + 7\dot{x} = \ddot{u} + 7\dot{u} + 5u \qquad \begin{array}{ll} \ddot{x}(0)=2 & \dot{u}(0)=0 \\ \dot{x}(0)=1 & u(0)=0 \\ x(0)=9 & \end{array}$$

Aplicando a transformada de Laplace:

$$\left(s^3 X(s) - \ddot{x}(0) - s\dot{x}(0) - s^2 x(0)\right) + 2\left(s^2 X(s) - \dot{x}(0) - s x(0)\right) +$$
$$7\left(s X(s) - x(0)\right) = \left(s^2 U(s) - s\,\cancel{u(0)}^{0} - \cancel{\dot{u}(0)}^{0}\right) + 7\left(s U(s) - \cancel{u(0)}^{0}\right) + 5U(s)$$

$$s^3 X(s) + 2s^2 X(s) + 7s X(s) - 2 - s - 9s^2 - 2 - 18s - 63 = s^2 U(s) + 7sU(s) + 5U(s)$$

$$X(s)\left(s^3 + 2s^2 + 7s\right) = U(s)\left(s^2 + 7s + 5\right) + 9s^2 + 19s + 67$$

$$X(s) = U(s)\,\frac{(s^2+7s+5)}{(s^3+2s^2+7s)} + \frac{9s^2+19s+67}{s^3+2s^2+7s}$$

OBS: $u(t) = 1 \rightarrow U(s) = \frac{1}{s}$

$$X(s) = \frac{s^2 + 7s + 5 + s(9s^2+19s+67)}{s^2(s^2+2s+7)}$$

$$\Downarrow$$

$$X(s) = \frac{9s^3 + 20s^2 + 74s + 5}{s^2(s^2+2s+7)}$$

• Resolvendo por frações parciais:

$$X(s) = \frac{508}{49s} + \frac{5}{7s^2} + \frac{-67s - 71}{49\left[(s+1)^2+6\right]} =$$

• Aplicando a transformada inversa:

$$X(t) = \frac{508}{49}\underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}}_{1} + \frac{5}{7}\underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}}_{t} - \frac{67}{49}\underbrace{\mathcal{L}^{-1}\left\{\frac{(s+1)}{(s+1)^2+6}\right\}}_{e^{-t}\cdot\cos(\sqrt{6}t)} - \frac{4}{49}\underbrace{\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+6}\right\}}_{\frac{1}{\sqrt{6}}e^{-t}\,\mathrm{sen}(\sqrt{6}t)}$$

$$\boxed{X(t) = \frac{508}{49} + \frac{5}{7}t - \frac{67}{49}\cdot e^{-t}\cos(\sqrt{6}t) - \frac{4}{49\sqrt{6}}\,e^{-t}\,\mathrm{sen}(\sqrt{6}t)}$$