PME 3380 - Modelagem de Sistemas Dinâmicos

Aula 20/10/20

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$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U$$

→ Laplace: $\begin{bmatrix} sX(s) - \underbrace{\cancel{X(0)}}^{0} \\ sY(s) - \underbrace{\cancel{Y(0)}}_{0} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U \Rightarrow$

$\Rightarrow \begin{bmatrix} sX(s) \\ sY(s) \end{bmatrix} = \begin{bmatrix} Y(s) \\ -100X(s) + 10U \end{bmatrix} \Rightarrow$

$\Rightarrow$ $$\boxed{X(s) = 10U / (s^2 + 100)}$$

$$\boxed{Y(s) = 10Us / (s^2 + 100)}$$

$\Rightarrow$ $$\boxed{FT(x) = \frac{X(s)}{U} = \frac{10}{(s^2 + 100)}}$$

$$\boxed{FT(y) = \frac{Y(s)}{U} = \frac{10s}{(s^2 + 100)}}$$

1.2-1

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix}}_{M} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} U$$

· Auto valor:

$$\det(M - \lambda I) = \begin{vmatrix} (-1-\lambda) & 4 & 0 \\ 5 & (2-\lambda) & 0 \\ -1 & 0 & (-3-\lambda) \end{vmatrix} =$$

$$= (1+\lambda)(3+\lambda)(2-\lambda) - 20(-3-\lambda) = 0 \Rightarrow$$

$$\Rightarrow (3+\lambda)\left[(1+\lambda)(2-\lambda) + 20\right] = 0 \Rightarrow$$

$$\Rightarrow \boxed{\lambda_1 = -3} \quad \boxed{\lambda_2 = -4{,}217} \quad \boxed{\lambda_3 = 5{,}217}$$

· Polos e auto valores coincidentes:

$$FT(x) = \frac{1}{(s-2)(s+1)-20} \; ; \; FT(y) = \frac{s+1}{(s-2)(s+1)-20}$$

2.1-1 L

$$\begin{cases} \ddot{x}_1 m_1 + K(x_1 - x_2) = u_1 \\ \ddot{x}_2 m_2 + K(x_2 - x_1) = u_2 \end{cases}$$

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/m_1 & 0 & K/m_1 & 0 \\ 0 & 0 & 0 & 1 \\ K/m_2 & 0 & -K/m_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1/m_1 & 0 \\ 0 & 0 \\ 0 & 1/m_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} s X_1 \\ s X_2 \\ s X_3 \\ s X_4 \end{bmatrix} = M \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + N \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$s X_1 = X_2 \quad ; \quad s X_3 = X_4$

$$s X_2 = \frac{-K}{m_1} X_1 + \frac{K}{m_1} X_3 + \frac{u_1}{m_1} = s^2 X_1$$

$$s X_4 = \frac{K}{m_2} X_1 - \frac{K}{m_2} X_3 + \frac{u_2}{m_2} = s^2 X_3$$

$$\left(s^2 + \frac{K}{m_2} \right) X_3 = \frac{K}{m_2} X_1 + \frac{u_2}{m_2}$$

$$\Rightarrow \quad X_1 = \frac{u_1/m_1 + K u_2/(m_1 m_2)}{s^2 \left(K/m_1 + K/m_2 + s^2 \right)}$$

$$\dot{X}_2 = \frac{-K(u_1 + u_2)/(m_1 m_2) - s^2 \cdot u_2/m_2}{s^2 \left(K/m_1 + K/m_2 + s^2 \right)}$$