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$$1.1) \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U$$

Fazendo a transformada de Laplas:

$$\begin{bmatrix} sX_{(s)} - X_{(0)} \\ sY_{(s)} - Y_{(0)} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} X_{(s)} \\ Y_{(s)} \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U_{(s)}$$

$$\left. \begin{array}{l} sX_{(s)} = Y_{(s)} \\ sY_{(s)} = -100X_{(s)} + 10U_{(s)} \end{array} \right\} Y_{(s)} = \frac{10sU_{(s)}}{s^2 + 100}$$

$$X_{(s)} = \frac{10U_{(s)}}{s^2 + 100}$$

$$\left\{ \begin{array}{l} F_T(y) = \frac{Y_{(s)}}{U_{(s)}} = \frac{10s}{s^2 + 100} \\ F_T(x) = \frac{X_{(s)}}{U_{(s)}} = \frac{10}{s^2 + 100} \end{array} \right.$$

$$1.2) \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} U$$

$$\det(A - \lambda I) = \begin{vmatrix} -1-\lambda & 4 & 0 \\ 5 & 2-\lambda & 0 \\ -1 & 0 & -3-\lambda \end{vmatrix} = -3-\lambda \left[(-1-\lambda)(2-\lambda) - 20 \right] + (3+5) \left[(-1-5)(2-5) - 20 \right]$$

ecce

Calculando raíces

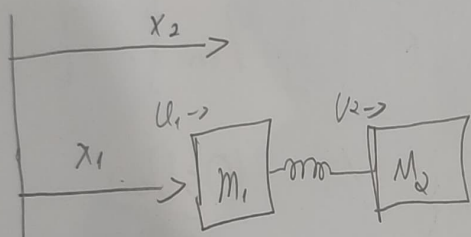
$$\begin{cases} s_1 = -3 \\ s_2 = -4,2 \\ s_3 = 5,2 \end{cases}$$

isto es, es polos e es auto-valores coincidentes

$$f_1(s) = \frac{s+1}{(s-2)(s+1)-20}$$

$$f_2(s) = \frac{1}{(s-2)(s+1)-20}$$

2.1)



$$m_1 \ddot{x}_1 = u_1 + k(x_2 - x_1)$$

$$m_2 \ddot{x}_2 = u_2 - k(x_2 - x_1)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -k/m_1 + k/m_2 & 0 \\ k/m_2 - k/m_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/m_1 & 0 \\ 0 & 1/m_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

-> Auto valores

$$\begin{vmatrix} -\lambda & 0 & 1 & 0 \\ 0 & -\lambda & 0 & 1 \\ -k/m_1 & k/m_1 & -\lambda & 0 \\ k/m_2 & -k/m_2 & 0 & -\lambda \end{vmatrix}$$

= Resolviendo el sistema

$$\lambda^2 = \frac{-\left(\frac{k}{m_1} + \frac{k}{m_2}\right) \pm \sqrt{\left(\frac{k}{m_1} + \frac{k}{m_2}\right)^2 - 4 \frac{k^2}{m_1 m_2}}}{2}$$