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PME3380

Exercício 1:

① $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u$

Usando laplace:

$$\begin{bmatrix} sX(s) - X(0) \\ sY(s) - Y(0) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U(s)$$

$$sX(s) = Y(s)$$
$$sY(s) = -100 \cdot X(s) + 10\,U(s)$$
$$\rightarrow Y(s) = \frac{10s \cdot U(s)}{s^2+100} \qquad X(s) = \frac{10 \cdot U(s)}{s^2+100}$$

Portanto, temos que: $F.T(y) = \frac{Y(s)}{U(s)} = \frac{10s}{s^2+100}$ // $\quad F.T(x) = \frac{X(s)}{U(s)} = \frac{10}{s^2+100}$ //

② $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u \longrightarrow$ domínio do tempo

$$\det(M-\lambda I) = \begin{vmatrix} -1-\lambda & 4 & 0 \\ 5 & 2-\lambda & 0 \\ -1 & 0 & -3-\lambda \end{vmatrix} = (-3-\lambda)\cdot\left[(2-\lambda)(-1-\lambda)-(4\cdot 5)\right] = (3+\lambda)(-\lambda^2+\lambda+22)$$

Portanto, temos:

$\boxed{\lambda = -3}$ $\boxed{\lambda = -4{,}22}$ $\boxed{\lambda = 5{,}22}$

$$\begin{bmatrix} sX(0) - X(0) \\ sY(0) - Y(0) \\ sZ(0) - Y(0) \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} X(s) \\ Y(s) \\ Z(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} U(0) \rightarrow$$

Domínio da Frequência

$$\begin{cases} s \cdot X(s) = -X(s) + 4Y(s) \\ sY(0) = 5X(s) + 2Y(s) \\ sY(s) = -X(s) - 3Z(s) \end{cases}$$

Portanto, temos:

$$\boxed{X(s) = \frac{4 \cdot U(s)}{s^2 - s - 22}} \qquad \boxed{Y(s) = \frac{(s+1) \cdot U(s)}{s^2 - s - 22}}$$

$$\boxed{Z(s) = \frac{-4U(s)}{(3+s)(s^2 - s - 22)}}$$

Por fim, na Função de transferência temos:

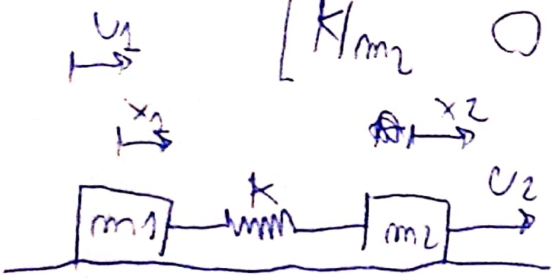
$$FT(x) = \frac{4}{s^2 - s - 22} \qquad FT(y) = \frac{(s+1)}{s^2 - s - 22} \qquad FT(z) = \frac{-4}{(3+s)(s^2 - s - 22)}$$

$$(3+s)(s^2 - s - 22) = 0 \longrightarrow \begin{array}{l} s_1 = -3 \\ s_2 = -4,22 \\ s_3 = 5,22 \end{array}$$

Exercício 3:

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k/m_1 & 0 & k/m_1 & 0 \\ 0 & 0 & 0 & 1 \\ k/m_2 & 0 & -k/m_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1/m_1 & 0 \\ 0 & 0 \\ 0 & 1/m_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\det(A-\lambda I) = \begin{vmatrix} -\lambda & 1 & 0 & 0 \\ -k/m_1 & -\lambda & k/m_1 & 0 \\ 0 & 0 & -\lambda & 1 \\ k/m_2 & 0 & -k/m_2 & -\lambda \end{vmatrix} = \lambda^2 + \frac{k\lambda^2}{m_2} + \frac{k\lambda^2}{m_1} = 0$$



$$\begin{cases} m_1\ddot{x}_1 + k(x_1 - x_2) = u_1 \\ m_2\ddot{x}_2 + k(x_2 - x_1) = u_2 \end{cases}$$

Resolvendo a equação anterior, temos: $\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 = \sqrt{\frac{k(m_1+m_2)}{m_1 m_2}}$, $\lambda_4 = -\sqrt{\frac{k(m_1+m_2)}{m_1 m_2}}$