Nome: Yago Neves Yamg NUSP: 10772626

*PME3380 - Lista do dia 20/10

1.1) Supondo que o sistema possa ser escrito da seguinte forma:

$$q = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} d \\ \dot{d} \end{bmatrix} \Rightarrow \dot{q} = \begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \end{bmatrix} = \begin{bmatrix} \dot{d} \\ \ddot{d} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} q + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u$$

$$\begin{cases} \dot{d}_1 = d_2 \\ \dot{d}_2 = -100 d_1 + 10u \end{cases} \xrightarrow[\text{Laplace}]{} \begin{cases} sD_1 - D_1(0) = D_2 \\ sD_2 - D_2(0) = -100D_1 + 10U \end{cases} \xrightarrow[\text{cond. iniciais nulas}]{} \begin{cases} sD_1 = D_2 \\ sD_2 = -100D_1 + 10U \end{cases}$$

Resolvendo para $D_1$: $s^2 D_1 + 100 D_1 = 10U \Rightarrow \boxed{D_1 = \frac{10U}{s^2 + 100}}$

já que $z = [1 \; 0] q + 0u$

Logo, $G(s) = \frac{D_1}{U} \Rightarrow \boxed{G(s) = \frac{10}{s^2+100}}$

---

1.2) A equação característica pode ser obtida a partir do polinômio característico da matriz A:

$$\det \begin{bmatrix} -1-w & 4 & 0 \\ 5 & 2-w & 0 \\ -1 & 0 & -3-w \end{bmatrix} \Rightarrow P_c(w) = -1.(0) + (-3-w).[(-1-w).(2-w) - 20]$$

$$P_c(w) = (-3-w).[(-1-w).(2-w) - 20]$$

Os autovalores são obtidos fazendo-se: $P_c(w) = 0 \Rightarrow (-3-w).[(-1-w).(2-w)-20] = 0$

Assim, os autovalores são obtidos de: $(-3-w) = 0 \Rightarrow \boxed{w_1 = -3}$

$$(-1-w).(2-w) - 20 = 0 \Rightarrow -2 - w + w^2 - 20 = 0 \Rightarrow$$

$$\Rightarrow w^2 - w - 22 = 0 \Rightarrow w = \frac{1 \pm \sqrt{1 - 4.1.(-22)}}{2} \Rightarrow$$

$$\Rightarrow \boxed{w_2 = \frac{1-\sqrt{89}}{2} \; ; \; w_3 = \frac{1+\sqrt{89}}{2}}$$

Supondo que o sistema possa ser escrito da seguinte forma:

$$q=\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}=\begin{bmatrix} d \\ \dot{d} \\ n \end{bmatrix} \Rightarrow \dot{q}=\begin{bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}=\begin{bmatrix} \dot{d} \\ \ddot{d} \\ \dot{n} \end{bmatrix}=\begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} q+\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$$

$$\begin{cases} \dot{d}_1 = -d_1 + 4d_2 \\ \dot{d}_2 = 5d_1 + 2d_2 + u \\ \dot{d}_3 = -d_1 - 3d_3 \end{cases} \xrightarrow[\text{Laplace}]{} \begin{cases} sD_1 - D_1(0) = -D_1 + 4D_2 \\ sD_2 - D_2(0) = 5D_1 + 2D_2 + U \\ sD_3 - D_3(0) = -D_1 - 3D_3 \end{cases} \xrightarrow[\text{Cond. iniciais nulas}]{} \begin{cases} sD_1 = -D_1 + 4D_2 \\ sD_2 = 5D_1 + 2D_2 + U \\ sD_3 = -D_1 - 3D_3 \end{cases}$$

Resolvendo para $D_1$ e $D_2$: $sD_2 = 5.\frac{4D_2}{s+1} + 2D_2 + U \Rightarrow s(s+1)D_2 = 20D_2 + 2(s+1)D_2 + (s+1)U$

$$D_2(s^2+s-2s-2-20) = (s+1)U$$

já que $z=\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} q + 0u$

$$\boxed{D_2 = \frac{(s+1)U}{s^2-s-22}}$$

$$D_1 = \frac{4\cancel{(s+1)}U}{s^2-s-22} \cdot \frac{1}{\cancel{(s+1)}} \Rightarrow \boxed{D_1 = \frac{4U}{s^2-s-22}}$$

$$\boxed{G_{D_2}(s) = \frac{D_2}{U} = \frac{s+1}{s^2-s-22}}$$

$$\boxed{G_{D_1}(s) = \frac{D_1}{U} = \frac{4}{s^2-s-22}}$$

Note que os polos vem de: $s^2-s-22=0 \rightarrow s_1 = \frac{1-\sqrt{89}}{2}$; $s_2 = \frac{1+\sqrt{89}}{2}$ que coincidem com dois dos autovalores calculados.

---

## 2.1) Da aula do dia 01/10:

$$x=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}=\begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} \Rightarrow \dot{x}=\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix}=\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}=\overbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K/M_1 & K/M_1 & 0 & 0 \\ K/M_2 & -K/M_2 & 0 & 0 \end{bmatrix}}^{A} x+\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/M_1 & 0 \\ 0 & 1/M_2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\Rightarrow \det(A-wI) = P_c(w) = \frac{1}{M_1M_2}\left[M_1M_2w^4 + K(M_1+M_2)w^2\right] \Rightarrow P_c(w) = w^4 + \frac{K(M_1+M_2)}{M_1M_2}w^2$$

$\Rightarrow$ autovalores: $w^2\left[w^2 + \frac{K(M_1+M_2)}{M_1M_2}\right] = 0 \Rightarrow w_1 = w_2 = 0$; $w_3 = \sqrt{-\frac{K(M_1+M_2)}{M_1M_2}}$

$$w_4 = -\sqrt{-\frac{K(M_1+M_2)}{M_1M_2}}$$

## 2.1) cont.

$$\begin{cases}\dot{x}_1 = x_3\\ \dot{x}_2 = x_4\\ \dot{x}_3 = -\frac{K}{M_1}x_1 + \frac{K}{M_1}x_2 + \frac{u_1}{M_1}\\ \dot{x}_4 = \frac{K}{M_2}x_1 - \frac{K}{M_2}x_2 + \frac{u_2}{M_2}\end{cases} \xrightarrow{\text{Laplace}} \begin{cases} sX_1 - X_1(0) = X_3\\ sX_2 - X_2(0) = X_4\\ sX_3 - X_3(0) = -\frac{K}{M_1}X_1 + \frac{K}{M_1}X_2 + \frac{U_1}{M_1}\\ sX_4 - X_4(0) = \frac{K}{M_2}X_1 - \frac{K}{M_2}X_2 + \frac{U_2}{M_2}\end{cases} \xrightarrow{\text{Cond. iniciais nulas}} \begin{cases} sX_1 = X_3\\ sX_2 = X_4\\ sX_3 = -\frac{K}{M_1}X_1 + \frac{K}{M_1}X_2 + \frac{U_1}{M_1}\\ sX_4 = \frac{K}{M_2}X_1 - \frac{K}{M_2}X_2 + \frac{U_2}{M_2}\end{cases}$$

→ Resolvendo para $X_1$ e $X_2$:

$$\begin{cases} s^2X_1 = -\frac{K}{M_1}(X_1 - X_2) + \frac{U_1}{M_1}\\ s^2X_2 = \frac{K}{M_2}(X_1 - X_2) + \frac{U_2}{M_2}\end{cases}$$

$$\rightarrow X_1 = \frac{U_1M_2s^2 + K(U_1+U_2)}{M_1M_2s^4 + K(M_1+M_2)s^2} \; ; \; X_2 = \frac{U_2M_1s^2 + K(U_1+U_2)}{M_1M_2s^4 + K(M_1+M_2)s^2}$$

polos: $M_1M_2s^4 + K(M_1+M_2)s^2 = 0 \Rightarrow s^2\left[M_1M_2s^2 + K(M_1+M_2)\right] = 0$

$s_1 = s_2 = 0$

$s_3 = \sqrt{-\frac{K(M_1+M_2)}{M_1M_2}} \; ; \; s_4 = -\sqrt{-\frac{K(M_1+M_2)}{M_1M_2}}$

Note que dividindo $X_1, X_2$ pela entrada, o denominador seria o mesmo.

---

## 2.2)

$$z = \begin{bmatrix} x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} = \begin{bmatrix} \bar{x}\\ \delta\\ \dot{\bar{x}}\\ \dot{\delta}\end{bmatrix} \Rightarrow \dot{z} = \begin{bmatrix} \dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} \dot{\bar{x}}\\ \dot{\delta}\\ \ddot{\bar{x}}\\ \ddot{\delta}\end{bmatrix} = \overbrace{\begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\\ 0 & -KM/M_1M_2 & 0 & 0\end{bmatrix}}^{A} z + \begin{bmatrix} 0 & 0\\ 0 & 0\\ 1/M & 1/M\\ 1/M_1 & -1/M_2\end{bmatrix}\begin{bmatrix} u_1\\ u_2\end{bmatrix}$$

$\det(A - wI) = P_c(w) = w^4 + \frac{KM}{M_1M_2}w^2 \Rightarrow$ autovalores: $w^2\left(w^2 + \frac{KM}{M_1M_2}\right) = 0$

$w_1 = w_2 = 0$

$w_3 = \sqrt{-\frac{KM}{M_1M_2}} \; ; \; w_4 = -\sqrt{-\frac{KM}{M_1M_2}}$

$$\begin{cases}\dot{x}_1 = x_3\\ \dot{x}_2 = x_4\\ \dot{x}_3 = \frac{u_1}{M} + \frac{u_2}{M}\\ \dot{x}_4 = -\frac{KM}{M_1M_2}x_2 + \frac{u_1}{M_1} - \frac{u_2}{M_2}\end{cases} \xrightarrow{\text{Laplace e cond. iniciais nulas}} \begin{cases} sX_1 = X_3\\ sX_2 = X_4\\ sX_3 = \frac{U_1}{M} + \frac{U_2}{M}\\ sX_4 = -\frac{KM}{M_1M_2}X_2 + \frac{U_1}{M_1} - \frac{U_2}{M_2}\end{cases} \xrightarrow{\text{Resolvendo para } X_1 \text{ e } X_2} \begin{cases} s^2X_1 = \frac{U_1}{M} + \frac{U_2}{M}\\ s^2X_2 = -\frac{KM}{M_1M_2}X_2 + \frac{U_1}{M_1} - \frac{U_2}{M_2}\end{cases}$$

$\rightarrow X_1 = \frac{U_1+U_2}{s^2M} \Rightarrow$ polos: $s_1 = s_2 = 0$ ; $X_2 = \frac{M_2U_1 - M_1U_2}{M_1M_2s^2 + KM} \rightarrow$ polos: $s_3 = \sqrt{-\frac{KM}{M_1M_2}}$ ; $s_4 = -\sqrt{-\frac{KM}{M_1M_2}}$