→ Determine as funções de transferência:

10769938
Wilson Siou Kan Chow

$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -100 & 0\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} + \begin{bmatrix}0\\ 10\end{bmatrix}u$$

→ Laplace

$$\begin{bmatrix}sX(s) - \cancel{X(0)}^{0}\\ sY(s) - \cancel{Y(0)}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -100 & 0\end{bmatrix}\begin{bmatrix}X(s)\\ Y(s)\end{bmatrix} + \begin{bmatrix}0\\ 10\end{bmatrix}U(s)$$

$$\begin{cases} sX(s) = Y(s) \\ sY(s) = -100X(s) + 10U(s) \end{cases}$$

$$\Rightarrow sY(s) = -100\frac{Y(s)}{s} + 10U(s) \rightarrow Y(s) = \frac{10s \cdot U(s)}{s^2+100}$$

$$X(s) = \frac{10U(s)}{s^2+100}$$

$$FT(x) = \frac{10}{s^2+100} \quad \text{e} \quad FT(y) = \frac{10s}{s^2+100}$$

1.2) $$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} = \begin{bmatrix}-1 & 4 & 0\\ 5 & 2 & 0\\ -1 & 0 & -3\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} + \begin{bmatrix}0\\ 1\\ 0\end{bmatrix}u$$

Domínio do Tempo

Solução da Homogênea:

$$\begin{cases} \dot{x} = -x+4y \\ \dot{y} = 5x+2y \\ \dot{z} = -x-3z \end{cases} \longrightarrow \begin{cases} x(t) = A\cdot e^{\lambda t} \\ y(t) = B\cdot e^{\lambda t} \\ z(t) = C\cdot e^{\lambda t} \end{cases} \Rightarrow \begin{bmatrix}A\lambda e^{t}\\ B\lambda e^{t}\\ C\lambda e^{t}\end{bmatrix} = \begin{bmatrix}-1 & 4 & 0\\ 5 & 2 & 0\\ -1 & 0 & 3\end{bmatrix}\begin{bmatrix}A e^{\lambda t}\\ B e^{\lambda t}\\ C e^{\lambda t}\end{bmatrix}$$

$$\begin{bmatrix}-1-\lambda & 4 & 0\\ 5 & 2-\lambda & 0\\ -1 & 0 & -3-\lambda\end{bmatrix}\begin{bmatrix}A\\ B\\ C\end{bmatrix} e^{\lambda t} = 0$$

Não quero a solução trivial.

$$\det\begin{bmatrix}-1-\lambda & 4 & 0\\ 5 & 2-\lambda & 0\\ -1 & 0 & -3-\lambda\end{bmatrix} = 0 \longrightarrow (-1-\lambda)(2-\lambda)(-3-\lambda) - 20(-3-\lambda) = 0$$

$$\lambda_1 = -3$$

$$(-1-\lambda)(2-\lambda) = 20 \longrightarrow \lambda_2 = \frac{1}{2} - \frac{\sqrt{89}}{2}$$

$$\lambda_3 = \frac{1}{2} + \frac{\sqrt{89}}{2}$$

Autovalores: -3; $\frac{1-\sqrt{89}}{2}$, $\frac{1+\sqrt{89}}{2}$

DOMÍNIO DA FREQUÊNCIA

— LAPLACE:

$$\begin{bmatrix} sX(s) - X(0) \\ sY(s) - Y(0) \\ sZ(s) - Z(0) \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} X(s) \\ Y(s) \\ Z(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} U(s)$$

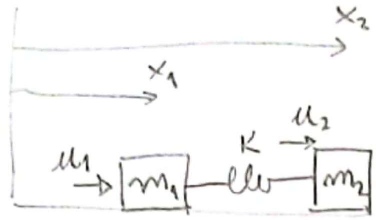
$$\left| \begin{array}{l} sX(s) = -X(s) + 4Y(s) \\ sY(s) = 5X(s) + 2Y(s) + U(s) \\ sZ(s) = -X(s) - 3Z(s) \end{array} \right. \Rightarrow$$

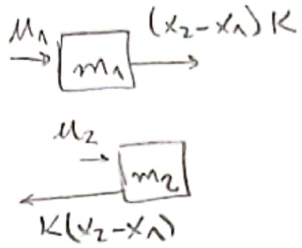
$$\Rightarrow \begin{pmatrix} X(s) = \dfrac{4U}{-22-s+s^2} \\ Y(s) = \dfrac{(1+s)U}{-22-s+s^2} \\ Z(s) = \dfrac{-4U}{(3+s)(-22-s+s^2)} \end{pmatrix} \qquad FT(s) = \begin{bmatrix} \dfrac{4}{s^2-s-22} \\ \dfrac{s+1}{s^2-s-22} \\ \dfrac{-4}{(3+s)(-22-s+s^2)} \end{bmatrix}$$

Polos: $(3+s)(-22-s+s^2) = 0$

$\hookrightarrow s_1 = -3 \; ; \; s_2 = \dfrac{1-\sqrt{89}}{2} \; ; \; s_3 = \dfrac{1+\sqrt{89}}{2}$ Os Polos e Autovalores ficaram iguais.

2)





$$m_2\ddot{x}_2 = u_2 - K(x_2-x_1)$$
$$m_1\ddot{x}_1 = u_1 + K(x_2-x_1)$$

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/m_1 & 0 & K/m_1 & 0 \\ 0 & 0 & 0 & 1 \\ K/m_2 & 0 & -K/m_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1/m_1 & 0 \\ 0 & 0 \\ 0 & 1/m_1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

Domínio do Tempo

$$\text{DET} \begin{vmatrix} -K & 1 & 0 & 0 \\ -K/m_1 & -\lambda & K/m_1 & 0 \\ 0 & 0 & -\lambda & 1 \\ K/m_2 & 0 & -K/m_2 & -\lambda \end{vmatrix} = 0 \rightarrow \lambda^4 + \frac{K^2}{m_1 m_2} - \frac{K^2}{m_1 m_2} = 0$$

$$\lambda^4 = 0 \rightarrow \lambda = 0$$

Domínio da Frequência

$$\begin{bmatrix} sX_1 - X_1(0) \\ s\dot{X}_1 - \dot{X}_1(0) \\ sX_2 - X_2(0) \\ s\dot{X}_2 - \dot{X}_2(0) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/m_1 & 0 & K/m_1 & 0 \\ 0 & 0 & 0 & 1 \\ K/m_2 & 0 & -K/m_2 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ \dot{X}_1 \\ X_2 \\ \dot{X}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ U_1/m_1 \\ 0 \\ U_2/m_2 \end{bmatrix} \rightarrow$$

$$sX_1 = \dot{X}_1$$
$$s\dot{X}_1 = \frac{-K}{m_1}X_1 + \frac{K}{m_1}X_2 + \frac{U_1}{m_1}$$
$$sX_2 = \dot{X}_2$$
$$s\dot{X}_2 = \frac{K}{m_2}X_1 - \frac{K}{m_2}X_2 + \frac{U_2}{m_2}$$

Resolvendo-

$$X_1 = \frac{\frac{-K}{m_2}\cdot\frac{U_1}{m_1} - s^2\frac{U_1}{m_1} - \frac{K}{m_1}\cdot\frac{U_2}{m_2}}{s^2\left(\frac{K}{m_1} + \frac{K}{m_2} + s^2\right)}$$

$$X_2 = \frac{\frac{-K}{m_2}\frac{U_1}{m_1} - \frac{K_1}{m_1}\frac{U_2}{m_2} - s^2\frac{U_2}{m_2}}{s^2\left(\frac{K}{m_1} + \frac{K}{m_2} + s^2\right)}$$

$$\dot{X}_1 = \frac{\frac{-K}{m_1}\frac{U_1}{m_1} - s^2\frac{U_1}{m_1} - \frac{K}{m_1}\frac{U_2}{m_1}}{s\left(\frac{K}{m_1} + \frac{K}{m_2} + s^2\right)}$$

$$\dot{X}_2 = \frac{\frac{-K}{m_2}\frac{U_1}{m_1} - \frac{K}{m_1}\frac{U_2}{m_2} - s^2\frac{U_2}{m_2}}{s\left(\frac{K}{m_1} + \frac{K}{m_2} + s^2\right)}$$

Rdos: $S_1 = 0$; $S_2 = 0$; $S_3 = \sqrt{\frac{K}{m_1} + \frac{K}{m_2}}\, i$;

$S_4 = -\sqrt{\frac{K}{m_1} + \frac{K}{m_2}}\, i$

2.2) O resultado é idêntico, mas:

$X_G = \frac{x_1 + x_2}{2}$ e $\delta = x_2 - x_1$