Exr. 20/10 - modelagem

Mariana Pin 9348644

1) $\left[\ \dot{X} = AX + BU\ \right]$

1.1) $\begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} X \\ \dot{X} \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U$

$\begin{bmatrix} sX(s) - \cancel{X(0)}^{\,0} \\ sY(s) - \cancel{Y(0)}^{\,0} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U$

$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = X \rightsquigarrow$ saída

$sX(s) = Y(s) + 0$

$sY(s) = -100\,X(s) + 10\,U$

$s^2 X = -100\,X + 10\,U$

$X(s^2 + 100) = 10\,U$

$\dfrac{X}{U} = \dfrac{10\,U}{s^2 + 100} \Rightarrow$ FT

* simulação última pg.

1.2) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} U$; $Y = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ x_2, x_1

$\left.\begin{aligned} s x_1 &= -x_1 + 4x_2 \\ s x_2 &= 5x_1 + 2x_2 + U \\ s x_3 &= -x_1 - 3x_3 \end{aligned}\right\}$

$x_1 = 4x_2/(s+1)$; $x_2 = x_1(s+1)/4$

$s x_2 = \dfrac{20 x_2}{s+1} + 2x_2 + u$

$\left(s - \dfrac{20}{s+1} - 2\right) x_2 = U \rightarrow \boxed{\dfrac{X_2}{U} = \dfrac{s+1}{s^2 - s + 22}}$

$\dfrac{X_2}{U} = \dfrac{x_1 \cancel{(s+1)}}{4u} = \dfrac{\cancel{s+1}}{s^2 - s - 22} \longrightarrow \boxed{\dfrac{X_1}{U} = \dfrac{4}{s^2 - s - 22}}$

Equação característica $\rightarrow (s^2 - s - 22)$

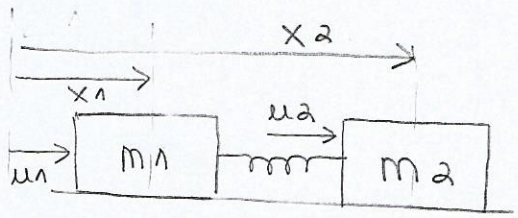
zeros: $s = -1$;

pólos $(s^2 - s - 22 = 0)$

↳ $s = 5{,}217$; $s = -4{,}217$

2)

sistema $\begin{cases} \ddot{x}_1 m_1 + k(x_1 - x_2) = u_1 \\ \ddot{x}_2 m_2 + k(x_2 - x_1) = u_2 \end{cases}$



$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k/m_1 & 0 & k/m_1 & 0 \\ 0 & 0 & 0 & 1 \\ k/m_2 & 0 & -k/m_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1/m_1 & 0 \\ 0 & 0 \\ 0 & 1/m_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

saídas: x_1 e $\dot{x}_2$

$$\begin{bmatrix} s x_1 \\ s x_2 \\ s x_3 \\ s x_4 \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$s x_2 = \frac{-k}{m_1} x_1 + \frac{k x_3}{m_1} + u_1/m_1 \longrightarrow s^2 x_1 = \frac{-k x_1}{m_1} + \frac{k x_3}{m_1} + \frac{u_1}{m_1}$$

$s x_1 = x_2$

$s x_3 = x_4$

$$s x_4 = \frac{k}{m_2} x_1 - \frac{k}{m_2} x_3 + u_2/m_2$$

$$s^2 x_3 = \frac{k}{m_2} x_1 - \frac{k}{m_2} x_3 + \frac{u_2}{m_2}$$

$$\left(s^2 + \frac{k}{m_2}\right) x_3 = \frac{k}{m_2} x_1 + \frac{u_2}{m_2}$$

$$x_1 = \frac{u_1/m_1 + k u_2/(m_1 \cdot m_2)}{s^2 (k/m_1 + k/m_2 + s^2)}$$

$$\dot{x}_2 = \frac{\frac{-k(u_1 + u_2)}{m_1 m_2} - s^2 u_2/m_2}{s^2 (k/m_1 + k/m_2 + s^2)}$$

pólos: $s = 0$; $s = 0$; $s = \pm\sqrt{k/m_1 + \frac{k}{m_2}}$

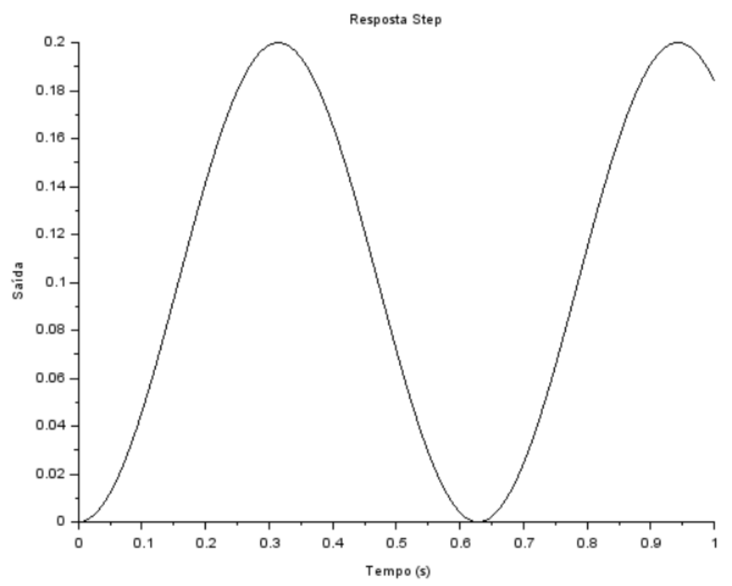
2.2)

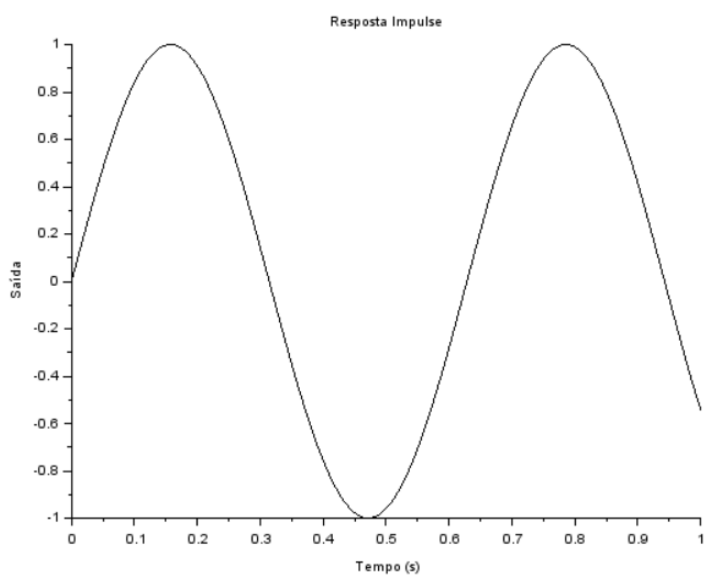
baricentro $z_1 = \frac{m_2 x_2 + m_1 x_1}{(m_1 + m_2)}$

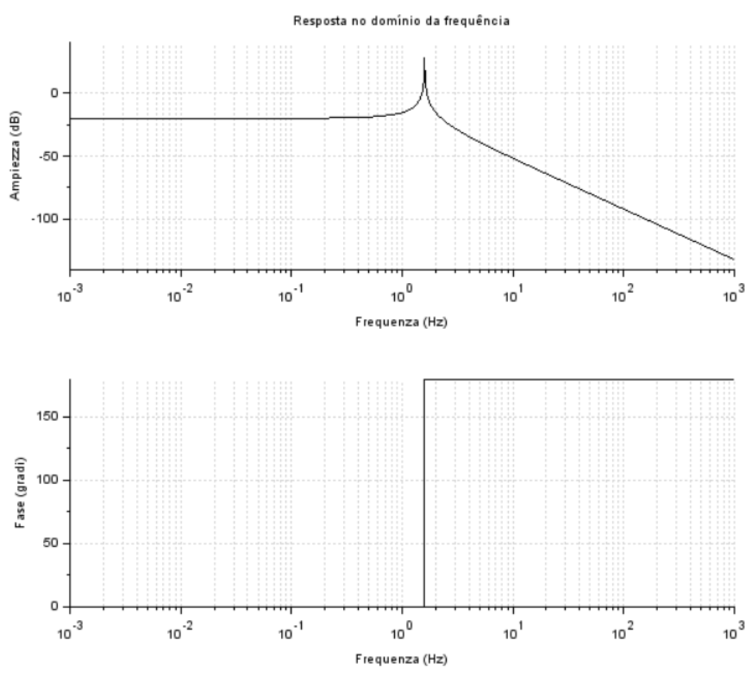
$\Delta = z_2 = x_2 - x_1 \longrightarrow \ddot{z}_2 = \ddot{x}_2 - \ddot{x}_1$

$$\ddot{x}_1 m_1 + \ddot{x}_2 m_2 = u_1 + u_2 \quad ; \quad \ddot{z}_2 = \frac{u_2 - k z_2}{m_2} - \frac{u_1 + k z_2}{m_1}$$

$$\ddot{z}_1 = \frac{u_1 + u_2}{(m_1 + m_2)}$$


Resposta Step
Saída
Tempo (s)
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1


Resposta Impulse
Saída
Tempo (s)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1


Resposta no domínio da frequência
Ampiezza (dB)
0
-50
-100
10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3}
Frequenza (Hz)
Fase (gradi)
150
100
50
0
10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3}
Frequenza (Hz)