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Modelagem Ex aula (20/10)

1.1) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u$

Transformada: $\begin{bmatrix} s\,X_1 - X_1(0) \\ s\,X_2 - X_2(0) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u$

$X_1(0) = X_2(0) = 0 \rightarrow \begin{bmatrix} sX_1 \\ sX_2 \end{bmatrix} =$ // //

$$\begin{cases} sX_1 = X_2 \\ sX_2 = -100X_1 + 10u \end{cases}$$

$$s^2 X_1 = 100X_1 + 10u$$

$$\frac{X_1}{u} = \frac{10}{s^2+100}$$

1.2) $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ $\quad y_1 = x_2 \,,\; y_2 = x_1$

$$\begin{vmatrix} -1-\lambda & 4 & 0 \\ 5 & 2-\lambda & 0 \\ -1 & 0 & -3-\lambda \end{vmatrix} = (-1-\lambda)(2-\lambda)(-3-\lambda) - [5 \cdot 4 \cdot (-3-\lambda)] = 0$$

$\lambda = -3$ $\quad (-1-\lambda)(2-\lambda) - 20 = 0$

$$\lambda = \frac{1 \pm \sqrt{89}}{2}$$

$$\begin{bmatrix} sx_1 \\ sx_2 \\ sx_3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$$

$$\begin{cases} sx_1 = -x_1 + 4x_2 \\ sx_2 = 5x_1 + 2x_2 + u \\ sx_3 = -x_1 - 3x_3 \end{cases}$$

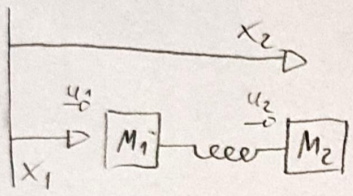
$$x_1 = \frac{4x_2}{s+1}$$

$$sx_2 = \frac{20x_2}{s+1} + 2x_2 + u$$

$$G_1 = \frac{x_2}{u} = \frac{(s+1)}{(s+1)(s-2)-20}$$

$$s = -1 \qquad s = \frac{1 \pm \sqrt{89}}{2}$$

2.1)



$$\begin{cases} m_1 \ddot{x}_1 = u_1 + K(x_2 - x_1) \\ m_2 \ddot{x}_2 = u_2 - K(x_2 - x_1) \end{cases}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} & & 1 & \\ & & & 1 \\ -K/m_1 & K/m_2 & & \\ K/m_2 & -K/m_1 & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/m_1 & 0 \\ 0 & 1/m_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{vmatrix} -\lambda & 0 & 1 & 0 \\ 0 & -\lambda & 0 & 1 \\ -K/m_1 & K/m_1 & -\lambda & 0 \\ K/m_2 & -K/m_2 & 0 & -\lambda \end{vmatrix} = \lambda^4 + \left(\frac{K}{m_1} + \frac{K}{m_2}\right)\lambda^2 + \frac{K^2}{m_1 m_2} = 0$$

$$\lambda^2 = \frac{-\left(\frac{K}{m_1} + \frac{K}{m_2}\right) \pm \sqrt{\left(\frac{K}{m_1} + \frac{K}{m_2}\right)^2 - 4\frac{K^2}{m_1 m_2}}}{2}$$

$$G = [C(sI - A)^{-1}B + D]$$