Samuel Alves 10769639

1.1 - $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u$

Usando a transformada de Laplace:

$$\begin{bmatrix} sX(s) - x(0) \\ sY(s) - y(0) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U(s)$$

$sX(s) = Y(s)$ $\quad$ $sY(s) = -100X(s) + 10U(s)$

$FT(x) = \frac{X(s)}{U(s)} = \frac{10}{s^2+100}$ $\quad$ $FT(y) = \frac{Y(s)}{U(s)} = \frac{10s}{s^2+100}$

1.2 - $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$

$\det(A - \lambda I) = \begin{bmatrix} -1-\lambda & 4 & 0 \\ 5 & 2-\lambda & 0 \\ -1 & 0 & -3-\lambda \end{bmatrix} \rightarrow = (-3-\lambda)[(-1-\lambda)(2-\lambda) - 20]$

$= (-3-s)[(-1-s)(2-s) - 20]$

$s_1 = -3$

$s_2 = -4{,}217$ $\quad$ $s_3 = 5{,}217$

$FT(x) = \frac{1}{(s-2)(s+1)+20}$ $\quad$ $FT(y) = \frac{s+1}{(s-2)(s+1)-20}$

2.1 $m_1\ddot{x}_1 - K(x_1 - x_2) = U_1$

$m_2\ddot{x}_2 - K(x_2 - x_1) = U_2$

$\dot{x}_1 = x_3$ $\quad$ $\dot{x}_3 = \frac{[U_1 + K(x_1 - x_2)]}{m_1}$

$\dot{x}_2 = x_4$ $\quad$ $\dot{x}_4 = \frac{[U_2 + K(x_2 - x_1)]}{m_2}$

$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K}{m_1} & -\frac{K}{m_1} & 0 & 0 \\ -\frac{K}{m_2} & \frac{K}{m_2} & 0 & 0 \end{bmatrix}$ $\quad$ $\det(A - \lambda I) = \begin{bmatrix} -\lambda & 0 & 1 & 0 \\ 0 & -\lambda & 0 & 1 \\ \frac{K}{m_1} & -\frac{K}{m_1} & -\lambda & 0 \\ -\frac{K}{m_2} & \frac{K}{m_2} & 0 & 0 \end{bmatrix}$

$s_1 = \lambda_1 = \sqrt{\frac{K}{m_2}}$ $\quad$ $s_2 = \lambda_2 = -\sqrt{\frac{K}{m_2}}$

$s_3 = \lambda_3 = \sqrt{\frac{K}{m_1}}$ $\quad$ $s_4 = \lambda_4 = \sqrt{\frac{K}{m_1}}$

2.2 - O resultado obtido será idêntico ao do exercício anterior.