Modelagem - aula 20/10

1.1

$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -100 & 0\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} + \begin{bmatrix}0\\ 10\end{bmatrix}u$$

$$\begin{cases} sX(s) = Y(s) \\ sY(s) = -100X(s) + 10U(s) \end{cases}$$

$$\mathcal{L} \Rightarrow \begin{bmatrix} s\cdot X(s) - \cancel{X(0)} \\ s\cdot Y(s) - \cancel{Y(0)} \end{bmatrix} = \begin{bmatrix}0 & 1\\ -100 & 0\end{bmatrix}\begin{bmatrix}X(s)\\ Y(s)\end{bmatrix} + \begin{bmatrix}0\\ 10\end{bmatrix}U(s)$$

$$\Rightarrow \begin{cases} X(s) = \dfrac{10\,U(s)}{100+s^2} = F_{tx}(s) \\ Y(s) = \dfrac{10\cdot s\cdot U(s)}{100+s^2} = F_{ty}(s) \end{cases}$$

1.2

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} = \begin{bmatrix}-1 & 4 & 0\\ 5 & 2 & 0\\ 4 & 0 & -3\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} + \begin{bmatrix}0\\ 1\\ 0\end{bmatrix}u$$

Autovalores da matriz A:

$\lambda_1 = -3 \quad \lambda_{2,3} = \dfrac{1 \pm \sqrt{89}}{2}$

Polos: -3, $\dfrac{1\pm\sqrt{89}}{2}$

$$\rightarrow \begin{aligned} x(t) &= \alpha_1 \cdot e^{\lambda_1 t} \\ y(t) &= \alpha_2 \cdot e^{\lambda_2 t} \\ z(t) &= \alpha_3 \cdot e^{\lambda_3 t} \end{aligned}$$

$\downarrow \mathcal{L}$

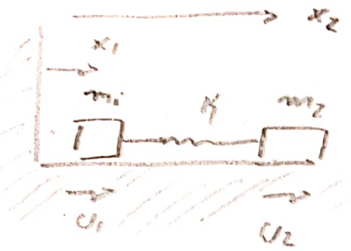
$$\begin{aligned} sX(s) &= -X(s) + 4Y(s) \\ sY(s) &= 5X(s) + 2Y(s) + U(s) \\ sZ(s) &= -X(s) - 3Z(s) \end{aligned} \Rightarrow$$

$X(s) = \dfrac{4U}{s^2-s-22} \Rightarrow FT(s) = \dfrac{4}{s^2-s-22}$

$Y(s) = \dfrac{(1+s)U}{s^2-s-22} \Rightarrow FT(y) \quad \dfrac{1+s}{s^2-s-22}$

$Z(s) = \dfrac{-4U}{(s+3)(s^2-s-22)} \Rightarrow FT(z) \quad \dfrac{-4}{(s+3)(s^2-s-22)}$

2.1



$$\begin{cases} m_2 \ddot{x}_2 = U_2 - K(x_2 - x_1) \\ m_1 \ddot{x}_1 = U_1 + K(x_2 - x_1) \end{cases}$$

$$\Downarrow$$

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/m_1 & 0 & K/m_1 & 0 \\ 0 & 0 & 0 & 1 \\ K/m_2 & 0 & -K/m_2 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1/m_1 & 0 \\ 0 & 0 \\ 0 & 1/m_2 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$\det(A - \lambda I) = \lambda^2 \left(\lambda^2 + K/m_1 + K/m_2\right) \quad \begin{cases} \lambda_1 = \lambda_2 = 0 \\ \lambda_3 = -\lambda_4 = \sqrt{-\left(\frac{K}{m_1} + \frac{K}{m_2}\right)} \end{cases}$$

$\mathcal{L}$

$$\begin{cases} sX_1 = \dot{X}_1 \\ s\dot{X}_1 = \frac{K(X_2 - X_1) + U_1}{m_1} \\ sX_2 = \dot{X}_2 \\ s\dot{X}_2 = \frac{K(X_1 - X_2) + U_2}{m_2} \end{cases} \Rightarrow$$

$$X_1(s) = \frac{\frac{-KU_1}{m_1 m_2} - s^2 \frac{U_1}{m_1} - \frac{KU_2}{m_1 m_2}}{s^2\left(\frac{K}{m_1} + \frac{K}{m_2} + s^2\right)} \qquad \dot{X}_1(s) = s X_1(s)$$

$$X_2(s) = \frac{\frac{-KU_1}{m_1 m_2} - \frac{s^2 U_2}{m_2} - \frac{KU_2}{m_1 m_2}}{s^2\left(\frac{K}{m_1} + \frac{K}{m_2} + s^2\right)} \qquad \dot{X}_2(s) = s X_2(s)$$

2.2 Uma mudança de coordenadas não altera o resultado do equacionamento e transformação