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PME3380 - Modelagem de Sistemas Dinâmicos

→ Exercício da Aula do dia 20/10/2020

1.1) $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u$

$$\Downarrow$$

$$\begin{bmatrix} sX_{(s)} \\ sY_{(s)} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} X_{(s)} \\ Y_{(s)} \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U_{(s)}$$

$$\Downarrow$$

$$\begin{cases} s X_{(s)} = Y_{(s)} & (1) \\ s Y_{(s)} = -100 X_{(s)} + 10 U_{(s)} & (2) \end{cases}$$

→ substituindo (1) em (2): $s \cdot s X_{(s)} = -100 X_{(s)} + 10 U_{(s)} \therefore X_{(s)} \cdot (s^2 + 100) = 10 U_{(s)}$

$$\therefore X_{(s)} = \frac{10 U_{(s)}}{(s^2+100)}$$

→ $s X_{(s)} = Y_{(s)}$ (1) → $s \cdot \frac{10 U_{(s)}}{(s^2+100)} = Y_{(s)} \therefore \quad Y_{(s)} = \frac{10 s U_{(s)}}{(s^2+100)}$

$$\rightarrow \begin{cases} FT_{(x)} = \dfrac{10}{(s^2+100)} \\ FT_{(y)} = \dfrac{10s}{(s^2+100)} \end{cases}$$

1.2) $\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} U$

→ Auto-valores : $\det(A-\lambda I) = \begin{bmatrix} -1-\lambda & 4 & 0 \\ 5 & 2-\lambda & 0 \\ -1 & 0 & -3-\lambda \end{bmatrix} = 0 \therefore (-1-\lambda)(2-\lambda)\cancel{(-3-\lambda)} - 20\cancel{(3-\lambda)} = 0$

$\lambda_1 = \frac{1-\sqrt{89}}{2}$; $\lambda_2 = -3$; $\lambda_3 = \frac{1+\sqrt{89}}{2}$

2.1) Equações: $\begin{cases} m_1 \ddot{X}_1 = U_1 - k(X_1 - X_2) \\ m_2 \ddot{X}_2 = U_2 - k(X_2 - X_1) \end{cases}$

→ $\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} X_3 \\ X_4 \\ \frac{U_1 + k(X_1 - X_2)}{m_1} \\ \frac{U_2 + k(X_2 - X_1)}{m_2} \end{bmatrix}$

→ $\det(A-\lambda I) = \begin{bmatrix} -\lambda & 0 & 1 & 0 \\ 0 & -\lambda & 0 & 1 \\ \frac{k}{m_1} & \frac{-k}{m_1} & -\lambda & 0 \\ \frac{-k}{m_2} & \frac{k}{m_2} & 0 & -\lambda \end{bmatrix} = 0 \therefore \lambda^4 + \frac{k^2}{m_1 m_2} - \frac{\lambda^2 k}{m_1} - \frac{\lambda^2 k}{m_2} = 0 \therefore$

$\therefore \lambda^2\left(\lambda^2 - \frac{k}{m_1} - \frac{k}{m_2}\right) + \frac{k^2}{m_1 m_2} = 0$

$\lambda_1 = -\sqrt{\frac{k}{m_1}}$; $\lambda_2 = -\sqrt{\frac{k}{m_2}}$; $\lambda_3 = \sqrt{\frac{k}{m_1}}$; $\lambda_4 = \sqrt{\frac{k}{m_2}}$