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Exercícios da Aula 20/10

1) Espaço de Estado:

$A=\begin{bmatrix}0 & 1\\ -100 & 0\end{bmatrix}$; $B=\begin{bmatrix}0\\ 10\end{bmatrix}$; $C=[1\ 0]$; $D=0$; $x(0)=[0\ 0]^T$; $\Delta t=0{,}1$; $t_f=1{,}0$

$sX_1=X_2$

$sX_2=-100X_1+10U \rightarrow sX_1=\frac{1}{s}(-100X_1+10U) \rightarrow X_1(s^2+100)=10U$

Para $Y=X_1$: $G(s)=\dfrac{Y(s)}{U(s)} \Rightarrow \boxed{G(s)=\dfrac{10}{s^2+100}}$

2) Espaço de Estado:

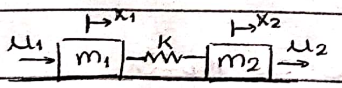
$A=\begin{bmatrix}-1 & 4 & 0\\ 5 & 2 & 0\\ -1 & 0 & -3\end{bmatrix}$; $B=\begin{bmatrix}0\\ 1\\ 0\end{bmatrix}$; $C=[0\ 0\ 1]$; $D=0$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}-1 & 4 & 0\\ 5 & 2 & 0\\ -1 & 0 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 1\\ 0\end{bmatrix}U \Rightarrow \begin{cases}sX_1=-X_1+4X_2\\ sX_2=5X_1+2X_2+U\\ sX_3=-X_1-3X_3\end{cases}$$

$$\det(A-\lambda I)=\begin{vmatrix}-1-\lambda & 4 & 0\\ 5 & 2-\lambda & 0\\ -1 & 0 & -3-\lambda\end{vmatrix}=(-1-\lambda)(2-\lambda)(-3-\lambda)-4\cdot5(-3-\lambda)=(-3-\lambda)((-1-s)(2-s)-20)$$

Eq. característica: $-s^3-2s^2+25s+66=0$

Polos: $s_1=-3$; $s_2=-4{,}217$; $s_3=5{,}217$

3) 

$$\begin{cases}m_1\ddot{x}_1+k(x_1-x_2)=u_1\\ m_2\ddot{x}_2+k(x_2-x_1)=u_2\end{cases}$$

$$\begin{bmatrix}\dot{x}_1\\ \ddot{x}_1\\ \dot{x}_2\\ \ddot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ -k/m_1 & 0 & k/m_1 & 0\\ 0 & 0 & 0 & 1\\ k/m_2 & 0 & -k/m_2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ \dot{x}_1\\ x_2\\ \dot{x}_2\end{bmatrix}+\begin{bmatrix}0 & 0\\ 1/m_1 & 0\\ 0 & 0\\ 0 & 1/m_2\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

$$\det(A-\lambda I)=\begin{vmatrix}-\lambda & 1 & 0 & 0\\ -k/m_1 & -\lambda & k/m_1 & 0\\ 0 & 0 & -\lambda & 1\\ k/m_2 & 0 & -k/m_2 & -\lambda\end{vmatrix}=\lambda^4+\frac{k\lambda^2}{m_2}+\frac{k\lambda^2}{m_1}=0$$

Autovalores: $\lambda_1=0$; $\lambda_2=0$; $\lambda_3=\sqrt{\frac{k}{m_1}+\frac{k}{m_2}}$; $\lambda_4=-\sqrt{\frac{k}{m_1}+\frac{k}{m_2}}$