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1.1) $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U$ transformada de Laplace

$$\begin{bmatrix} S X(s) - \cancel{X(0)} \\ S Y(s) - \cancel{Y(0)} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \cdot \begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U(s)$$

0

$$\begin{cases} S X(s) = Y(s) \\ S Y(s) = -100 X(s) + 10 U(s) \end{cases}$$

$$X(s) = \frac{10 U(s)}{s^2 + 100} \quad ; \quad Y(s) = \frac{S\, 10\, U(s)}{s^2 + 100}$$

$$\therefore \ FT(x) = \frac{X(s)}{U(s)} = \frac{10}{s^2 + 100} \ ; \ FT(y) = \frac{Y(s)}{U(s)} = \frac{S\,10}{s^2 + 100}$$

1.2) $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} U \rightarrow$

$$S X_1 = -1 X_1 + 4 X_2$$
$$S X_2 = 5 X_1 + 2 X_2 + U$$
$$S X_3 = -X_1 - 3 X_3$$

$$\det(A - \lambda I) = \begin{vmatrix} -1-\lambda & 4 & 0 \\ 5 & 2-\lambda & 0 \\ -1 & 0 & -3-\lambda \end{vmatrix} = (-3-\lambda)\left[(-1-\lambda)(2-\lambda) - 20\right] = 0$$

$$\begin{cases} \lambda_1 = -3 \\ \lambda_2 = \sqrt{89}/2 + 0{,}5 \approx 5{,}217 \\ \lambda_3 = -\sqrt{89}/2 + 0{,}5 \approx -4{,}217 \end{cases}$$

$$FT(x) = \frac{1}{(s-2)(s+1) - 20} \quad ; \quad FT(y) = \frac{s+1}{(s-2)(s+1) - 20}$$

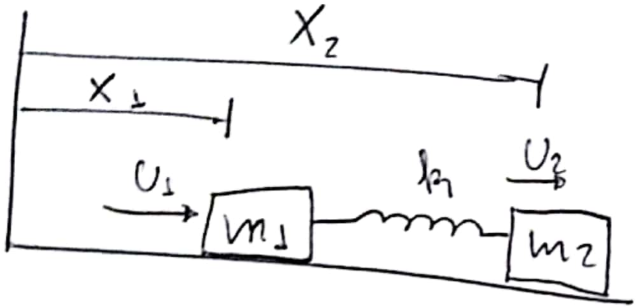
Zeros: s = indefinido

Polos: $S_1 = \lambda_3$
$S_2 = \lambda_2$

Zeros: $S = -1$

Polos: $S_1 = \lambda_3$
$S_2 = \lambda_2$

2.1)



$$\begin{cases} m_1\ddot{X}_1 - k(X_1 - X_2) = U_1 \\ m_2\ddot{X}_2 - k(X_2 - X_1) = U_2 \end{cases}$$

Espaço de Estados

$$\begin{matrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{matrix} \Rightarrow \begin{matrix} \dot{X}_1 = X_3 \\ \dot{X}_2 = X_4 \\ \dot{X}_3 = [U_1 + k(X_1 - X_2)]/m_1 \\ \dot{X}_4 = [U_2 + k(X_2 - X_1)/m_2 \end{matrix}$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ k/m_1 & -k/m_1 & 0 & 0 \\ -k/m_2 & k/m_2 & 0 & 0 \end{bmatrix}$$

$$\det(A - \lambda I) = 0 \longrightarrow \lambda^4 - \frac{k^2}{m_1 m_2} - \lambda^2\left(\frac{k}{m_2} + \frac{k}{m_1}\right) = 0$$

Raízes: $\lambda_1 = \sqrt{\frac{k}{m_2}}$; $\lambda_2 = -\lambda_1$; $\lambda_3 = \sqrt{\frac{k}{m_1}} = \lambda_4$