Gabriel Barbosa Paganini - 10772539 - Modelagem aula 20/10

1.1) $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u \xrightarrow{\text{Laplace}} \begin{bmatrix} sX(s) - \cancel{X(0)} \\ sY(s) - \cancel{Y(0)} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U(s)$

$\begin{cases} sX(s) = Y(s) \\ sY(s) = -100\,X(s) + 10\,U(s) \end{cases} \longrightarrow \boxed{X(s) = \dfrac{10\,U(s)}{100+s^2}}$ e $\boxed{Y(s) = \dfrac{10s\,U(s)}{100+s^2}}$

. Função de Transferência : $\boxed{FT(x) = \dfrac{10}{100+s^2}}$ e $\boxed{FT(y) = \dfrac{10s}{100+s^2}}$

1.2) $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \underbrace{\begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix}}_{M} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$ → . Domínio do tempo : achar autovalores

$\det(M - \lambda I) = \begin{vmatrix} -1-\lambda & 4 & 0 \\ 5 & 2-\lambda & 0 \\ -1 & 0 & -3-\lambda \end{vmatrix} = (-3-\lambda)\cdot[(2-\lambda)\cdot(-1-\lambda) - 4\cdot 5] = (3+\lambda)\cdot(-\lambda^2+\lambda+22)$

$\boxed{\lambda_1 = -3}$; $\boxed{\lambda_2 = -4{,}22}$; $\boxed{\lambda_3 = 5{,}22}$

. Domínio da frequência : $\begin{bmatrix} sX(s) - \cancel{X(0)} \\ sY(s) - \cancel{Y(0)} \\ sZ(s) - \cancel{Z(0)} \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} X(s) \\ Y(s) \\ Z(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} U(s)$

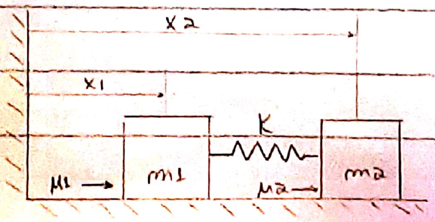
$\begin{cases} sX(s) = -X(s) + 4\,Y(s) \\ sY(s) = 5\,X(s) + 2\,Y(s) + U(s) \\ sZ(s) = -X(s) - 3\,Z(s) \end{cases} \longrightarrow \boxed{X(s) = \dfrac{4\,U(s)}{s^2-s-22}}$; $\boxed{Y(s) = \dfrac{(s+1)\,U(s)}{s^2-s-22}}$

$\boxed{Z(s) = \dfrac{-4\,U(s)}{(3+s)\cdot(s^2-s-22)}}$

. Função de transferência : $\boxed{FT(x) = \dfrac{4}{s^2-s-22}}$ $\boxed{FT(y) = \dfrac{(s+1)}{s^2-s-22}}$ $\boxed{FT(z) = \dfrac{-4}{(3+s)(s^2-s-22)}}$

. Polos : $(3+s)\cdot(s^2-s-22) = 0 \longrightarrow \boxed{s_1 = -3}$; $\boxed{s_2 = -4{,}22}$; $\boxed{s_3 = 5{,}22}$

↳ bateram com os autovalores !

2) 

$\begin{cases} m_1\,\ddot{x}_1 = u_1 + K(x_2 - x_1) \\ m_2\,\ddot{x}_2 = u_2 - K(x_2 - x_1) \end{cases}$

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/m_1 & 0 & +K/m_1 & 0 \\ 0 & 0 & 0 & 1 \\ +K/m_2 & 0 & -K/m_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1/m_1 & 0 \\ 0 & 0 \\ 0 & 1/m_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

. Domínio do tempo:

$$\det \begin{vmatrix} -\lambda & 1 & 0 & 0 \\ -K/m_1 & -\lambda & +K/m_1 & 0 \\ 0 & 0 & -\lambda & 1 \\ +K/m_2 & 0 & -K/m_2 & -\lambda \end{vmatrix} = \lambda^2 \cdot \left(-\lambda^2 + K/m_1 + K/m_2\right)$$

$$\boxed{\lambda_1 = 0 \quad \lambda_2 = 0} \qquad \boxed{\lambda_{3,4} = \pm\sqrt{\frac{-K(m_1+m_2)}{m_1 m_2}}}$$

. Domínio da frequência:

$$\begin{bmatrix} sX_1(s) - \cancel{X_1(0)} \\ s\dot{X}_1(s) - \cancel{\dot{X}_1(0)} \\ sX_2(s) - \cancel{X_2(0)} \\ s\dot{X}_2(s) - \cancel{\dot{X}_2(0)} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/m_1 & 0 & K/m_1 & 0 \\ 0 & 0 & 0 & 1 \\ K/m_2 & 0 & -K/m_2 & 0 \end{bmatrix} \begin{bmatrix} X_1(s) \\ \dot{X}_1(s) \\ X_2(s) \\ \dot{X}_2(s) \end{bmatrix} + \begin{bmatrix} 0 \\ U_1/m_1 \\ 0 \\ U_2/m_2 \end{bmatrix}$$

$$\begin{cases} sX_1 = \dot{X}_1 \\ s\dot{X}_1 = -KX_1/m_1 + KX_2/m_1 + U_1/m_1 \\ sX_2 = \dot{X}_2 \\ s\dot{X}_2 = KX_1/m_2 - KX_2/m_2 + U_2/m_2 \end{cases}$$

$$\rightarrow \boxed{X_1 = \frac{-\frac{KU_1}{m_1 m_2} - \frac{s^2 U_1}{m_1} - \frac{KU_2}{m_1 m_2}}{s^2\left(\frac{K}{m_1} + \frac{K}{m_2} + s^2\right)}}$$

$$\rightarrow \boxed{\dot{X}_1 = \frac{-\frac{KU_1}{m_1 m_2} - \frac{s^2 U_1}{m_1} - \frac{KU_2}{m_1 m_2}}{s\left(\frac{K}{m_1} + \frac{K}{m_2} + s^2\right)}}$$

$$\rightarrow \boxed{X_2 = \frac{-\frac{K(U_1+U_2)}{m_1 m_2} - \frac{s^2 U_2}{m_2}}{s^2\left(\frac{K}{m_1} + \frac{K}{m_2} + s^2\right)}} \quad ; \quad \rightarrow \boxed{\dot{X}_2 = \frac{-\frac{K(U_1+U_2)}{m_1 m_2} - \frac{s^2 U_2}{m_2}}{s\left(\frac{K}{m_1} + \frac{K}{m_2} + s^2\right)}}$$

. Polos: $s^2\left(\frac{K}{m_1} + \frac{K}{m_2} + s^2\right) = 0 \rightarrow \boxed{s_1 = 0, \; s_2 = 0, \; s_{3,4} = \pm\sqrt{\frac{-K(m_1+m_2)}{m_1 m_2}}}$

$\rightarrow$ batem com os autovalores!