ITALO PAIVA - 10853310 - exercício 20/10

① $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u$

transformada de laplace

$$\begin{bmatrix} sx(s) - x(0) \\ sy(s) - y(0) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} x(s) \\ y(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U(s)$$

$sx(s) = y(s)$, $sy(s) = -100\,x(s) + 10\,U(s)$

$\therefore FT(y) = \dfrac{y(s)}{U(s)} = \dfrac{10s}{s^2+100}$; $FT(x) = \dfrac{X(s)}{U(s)} = \dfrac{10}{s^2+100}$

1.2) $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \begin{matrix} x \\ y \\ z \end{matrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} U$

$\det(A - \lambda I) = \begin{bmatrix} -1-\lambda & 4 & 0 \\ 5 & 2-\lambda & 0 \\ -1 & 0 & -3-\lambda \end{bmatrix}$ $\therefore$ tem-se:

$\lambda_1 = -3$

$\lambda_2 = -4{,}217$

$\lambda_3 = 5{,}21$

$\therefore F_T(y) = \dfrac{s+1}{(s-2)(s+1)-20}$, $F_T(x) = \dfrac{1}{(s-2)(s+1)-20}$

os polos e os autovalores coincidiram

(2) $m_1 \ddot{x}_1 - K(x_1 - x_2) = U_1$.

$m_2 \ddot{x}_2 - K(x_2 - x_1) = U_2$

$x_1 \qquad \dot{x}_1 = x_3$

$x_2 \quad \Rightarrow \quad \dot{x}_2 = x_4$

$x_3 \qquad \dot{x}_3 = [U_1 + K(x_1 - x_2)]/m_1$

$x_4 \qquad \dot{x}_4 = [U_2 + K(x_2 - x_1)]/m_2$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ K/m_1 & -K/m_1 & 0 & 0 \\ -K/m_2 & K/m_2 & 0 & 0 \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{bmatrix} -\lambda & 0 & 1 & 0 \\ 0 & -\lambda & 0 & 1 \\ K/m_1 & -K/m_1 & -\lambda & 0 \\ -K/m_2 & K/m_2 & 0 & 0 \end{bmatrix} \rightarrow \lambda^4 + \frac{K^2}{m_1 + m} - \lambda^2\left(\frac{K}{m_2} + \frac{K}{m_1}\right) = 0$$

raízes obtidas são

$\lambda_1 = \sqrt{\frac{K}{m_2}} \qquad \lambda_2 = -\sqrt{\frac{K}{m_2}} \qquad \lambda_3 = \sqrt{\frac{K}{m_1}} \qquad \lambda_4 = \sqrt{\frac{K}{m_1}}$