PME 3380 - Exc. 20/10

Gabriel Rodrigues Camargo - 10772460

(1)

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u$$

Transformada de Laplace:

$$\begin{bmatrix} sX(s) - X(0) \\ sY(s) - Y(0) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U(s)$$

$$\begin{cases} s\cancel{X}(s) = Y(s) \\ sY(s) = -100X(s) + 10U(s) \end{cases}$$

Resolvendo $\Rightarrow$ $X(s) = \dfrac{10U(s)}{s^2+100}$

$Y(s) = \dfrac{10\,s\,U(s)}{s^2+100}$

Logo $FT(x) = \dfrac{10}{s^2+100}$ e $FT(y) = \dfrac{10s}{s^2+100}$

(2)

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$$

- Domínio do tempo

$$\begin{cases} \dot{x} = -x + 4y \\ \dot{y} = 5x + 2y \\ \dot{z} = -x + 3z \end{cases} \xrightarrow{\text{Transf}} \begin{cases} x(t) = Ae^{\lambda t} \\ y(t) = Be^{\lambda t} \\ z(t) = Ce^{\lambda t} \end{cases}$$

$$\begin{bmatrix} A\lambda e^{\lambda t} \\ B\lambda e^{\lambda t} \\ C\lambda e^{\lambda t} \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} Ae^{\lambda t} \\ Be^{\lambda t} \\ Ce^{\lambda t} \end{bmatrix}$$

Para achar sol homogênea;

$$\det\begin{bmatrix} -1-\lambda & 4 & 0 \\ 5 & 2-\lambda & 0 \\ -1 & 0 & -3-\lambda \end{bmatrix} = 0 \Rightarrow (-1-\lambda)(2-\lambda)(-3-\lambda) - 20(-3-\lambda) = 0$$

$$\lambda_1 = -3$$

$$\lambda_2 = \frac{1}{2} + \frac{\sqrt{89}}{2}$$

$$\lambda_3 = \frac{1}{2} - \frac{\sqrt{89}}{2}$$

Autovalores!

- Domínio da frequência

Transf Laplace:

$$\begin{bmatrix} sX(s) - \cancel{X(0)} \\ sY(s) - \cancel{Y(0)} \\ sZ(s) - \cancel{Z(0)} \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} X(s) \\ Y(s) \\ Z(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} U(s)$$

$$\begin{cases} sX(s) = -X(s) + 4Y(s) \\ sY(s) = 5X(s) + 2Y(s) + U(s) \\ sZ(s) = X(s) - 3Z(s) \end{cases} \rightarrow \begin{cases} X(s) = 4U/(-22-s+s^2) \\ Y(s) = \dfrac{(1+s)U}{-22-s+s^2} \\ Z(s) = \dfrac{-4U}{(3+s)(-22-s+s^2)} \end{cases}$$

$$FT(s) = \begin{bmatrix} \dfrac{4}{s^2-s-22} \\ \dfrac{s+1}{s^2-s-22} \\ \dfrac{-4}{(3+s)(-22-s+s^2)} \end{bmatrix} \rightarrow$$

Polos:

$$(3+s)(-22-s+s^2) = 0$$

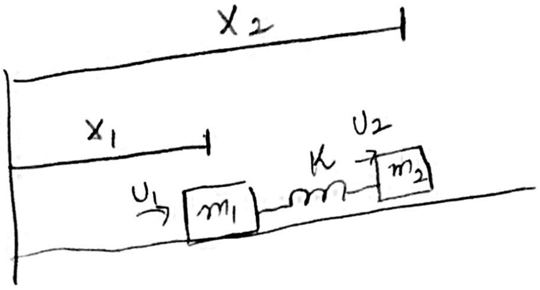
$$s_1 = -3$$

$$s_2 = \frac{1+\sqrt{89}}{2}$$

$$s_3 = \frac{1-\sqrt{89}}{2}$$

→ Polos iguais!

③



$$m_2 \ddot{x}_2 = U_2 - K(X_2 - X_1)$$

$$m_1 \ddot{X}_1 = U_1 + K(X_2 - X_1)$$

$$\begin{bmatrix} \dot{X}_1 \\ \ddot{X}_1 \\ \dot{X}_2 \\ \ddot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/m_1 & 0 & K/m_1 & 0 \\ 0 & 0 & 0 & 1 \\ K/m_2 & 0 & -K/m_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1/m_1 & 0 \\ 0 & 0 \\ 0 & 1/m_2 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

- Domínio temporal

$$\begin{vmatrix} -\lambda & 1 & 0 & 0 \\ -K/m_1 & -\lambda & K/m_1 & 0 \\ 0 & 0 & -\lambda & 1 \\ K/m_2 & 0 & -K/m_2 & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^4 + \frac{K^2}{m_1 + m_2} - \frac{K^2}{m_1 + m_2}$$

$$\lambda = 0$$

- Domínio frequência

Transformada

$$\begin{bmatrix} s x_1 - X(0) \\ s\dot{x}_1 - \dot{x}_1(0) \\ sX_2 - x_2(0) \\ s\dot{x}_2 - \dot{x}_2(0) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/m_1 & 0 & K/m_1 & 0 \\ 0 & 0 & 0 & 1 \\ K/m_2 & 0 & -K/m_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ U_1/m_1 \\ 0 \\ U_2/m_2 \end{bmatrix} =$$

Resolvendo

$$X_1 = \frac{\dfrac{-K U_1}{m_2 m_1} - \dfrac{s^2 U_1}{m_1} - \dfrac{K U_2}{m_1 m_2}}{s^2\left(\dfrac{K}{m_1} + \dfrac{K}{m_2} + s^2\right)} \qquad X_2 = \frac{\dfrac{-K U_1}{m_2 m_1} - \dfrac{K_1 U_2}{m_1 m_2} - \dfrac{s^2 U_2}{m_2}}{s^2\left(\dfrac{K}{m_1} + \dfrac{K}{m_2} + s^2\right)}$$

$\dot{x}_1 = s x_1$ e $\dot{x}_2 = s x_2$

Polos $s_1 = s_2 = 0$ e $s_3 = \sqrt{\frac{k_1}{m_1} + \frac{k}{m_2}}\, i$ e $s_4 = -\sqrt{\frac{k}{m_1} + \frac{k}{m_2}}\, i$

3.2 O uso das variáveis $x_G = \frac{x_1 + x_2}{2}$ e $\delta = x_2 - x_1$ será o mesmo porque a mudança de variável não implica em mudança da característica do sistema.