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Disciplina: PME3380 - Modelagem de Sistemas Dinâmicos

1.1) $$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U$$

- Aplicando a transformada de Laplace:

$$\begin{bmatrix} sX(s) - X(0) \\ sY(s) - Y(0) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U(s)$$

$sX(s) = Y(s)$
$sY(s) = -100X(s) + 10U(s)$

$$\therefore FT(y) = \frac{Y(s)}{U(s)} = \frac{10s}{s^2+100} \qquad FT(x) = \frac{X(s)}{U(s)} = \frac{10}{s^2+100}$$

1.2) $$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} U$$

$$\det(A - \lambda I) = \begin{vmatrix} -1-\lambda & 4 & 0 \\ 5 & 2-\lambda & 0 \\ -1 & 0 & -3-\lambda \end{vmatrix}$$

$$\det(A - \lambda I) = (-3-\lambda)[(-1-\lambda)(2-\lambda) - 20]$$

- Sendo $\lambda = 5$ temos que:

- $s_1 = -3$
- $s_2 = -4{,}217$
- $s_3 = 5{,}217$

$$\therefore FT(y) = \frac{s+1}{(s-2)(s+1)-20} \qquad FT(x) = \frac{1}{(s-2)(s+1)-20}$$

2.1)

$$\begin{cases} m_1\ddot{x}_1 - k(x_1 - x_2) = U_1 \\ m_2\ddot{x}_2 - k(x_2 - x_1) = U_2 \end{cases}$$

$$\dot{x}_1 = x_3$$
$$\dot{x}_2 = x_4$$
$$\dot{x}_3 = \frac{[U_1 + k(x_1 - x_2)]}{m_1}$$
$$\dot{x}_4 = \frac{[U_2 + k(x_2 - x_1)]}{m_2}$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ k/m_1 & -k/m_1 & 0 & 0 \\ -k/m_2 & k/m_2 & 0 & 0 \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} -\lambda & 0 & 1 & 0 \\ 0 & -\lambda & 0 & 1 \\ k/m_1 & -k/m_2 & -\lambda & 0 \\ -k/m_2 & k/m_2 & 0 & 0 \end{vmatrix}$$

$$\therefore s_1 = \lambda_1 = \sqrt{\frac{k}{m_2}} \qquad s_2 = \lambda_2 = -\sqrt{\frac{k}{m_2}}$$

$$s_3 = \lambda_3 = \sqrt{\frac{k}{m_1}} \qquad s_4 = \lambda_4 = \sqrt{\frac{k}{m_1}}$$

2.2) O resultado será idêntico ao do exercício anterior