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1.1) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u$

↳ Transformada

$$\begin{bmatrix} sX_1 - x_1(0) \\ sX_2 - x_2(0) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U$$

$X_1(0) = X_2(0) = 0$

$$\begin{bmatrix} sX_1 \\ sX_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U$$

$$\begin{cases} sX_1 = X_2 & \text{I} \\ sX_2 = -100X_1 + 10U & \text{II} \end{cases}$$

↳ Saída $= X_1$; entrada $= U$ I em II

$$s^2 X_1 = -100X_1 + 10U$$

$$\frac{X_1}{U} = \frac{10}{s^2+100}$$

1.2) ↳ Determinando as saídas

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \Rightarrow \begin{matrix} Y_1 = X_2 \\ Y_2 = X_1 \end{matrix}$$

↳ Autovalores e eq. característica

$$\begin{vmatrix} -1-\lambda & 4 & 0 \\ 5 & 2-\lambda & 0 \\ -1 & 0 & -3-\lambda \end{vmatrix} \Rightarrow (-1-\lambda)(2-\lambda)(-3-\lambda) - 20(-3-\lambda) = 0$$

$$\lambda = -3$$

$$(-1-\lambda)(2-\lambda) - 20 = 0$$

$$\lambda = \frac{1 \pm \sqrt{89}}{2}$$

↳ Determinando as FTs

$$\begin{bmatrix} sX_1 \\ sX_2 \\ sX_3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} U$$

$$\begin{cases} sX_1 = -X_1 + 4X_2 & \text{I} \\ sX_2 = 5X_1 + 2X_2 + U & \text{II} \\ sX_3 = -X_1 - 3X_3 & \text{III} \end{cases}$$

↳ Por I

$$X_1 = \frac{4}{s+1} X_2 \quad \text{(IV)}$$

↳ IV em II

$$sX_2 = 5\left(\frac{4}{s+1}\right) X_2 + 2X_2 + U$$

$$s(s+1)X_2 = 20\,X_2 + 2(s+1)X_2 + (s+1)U$$

$$G_1 = \frac{X_2}{U} = \frac{(s+1)}{(s+1)(s-2)-20} \Rightarrow \text{zero} \Rightarrow s = -1 \quad \text{polos} \Rightarrow s = \frac{1 \pm \sqrt{89}}{2}$$

↳ IV em II

$$X_2 = \frac{s+1}{4} X_1$$

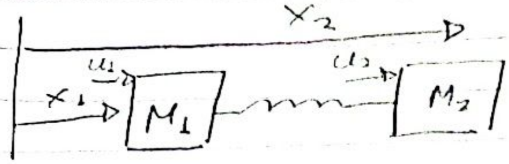
$$s\left(\frac{s+1}{4}\right) X_1 = 5X_1 + 2\left(\frac{s+1}{4}\right) X_1 + U$$

$$s(s+1)X_1 = 20X_1 + 2(s+1)X_1 + 4U$$

$$G_2 = \frac{X_1}{U} = \frac{4}{(s+1)(s-2)-20} \Rightarrow \text{zero} = \{\emptyset\} \quad \text{polos} = s = \frac{1 \pm \sqrt{89}}{2}$$

Os pólos são os Autovalores de A

2.1)



$$\begin{cases} m_1 \ddot{x}_1 = u_1 + K(x_2 - x_1) \\ m_2 \ddot{x}_2 = u_2 - K(x_2 - x_1) \end{cases}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} & & 1 & \\ & & & 1 \\ -K/m_1 & K/m_1 & & \\ K/m_2 & -K/m_2 & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/m_1 & \\ & 1/m_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

↳ Auto valores

$$\begin{vmatrix} -\lambda & 0 & 1 & 0 \\ 0 & -\lambda & 0 & 1 \\ -K/m_1 & K/m_1 & -\lambda & 0 \\ K/m_2 & -K/m_2 & 0 & -\lambda \end{vmatrix} = \lambda^4 + \frac{K^2}{m_1 m_2} + \lambda^2 \frac{K}{m_2} + \lambda^2 \frac{K}{m_1}$$

$$\lambda^4 + \left(\frac{K}{m_1} + \frac{K}{m_2}\right)\lambda^2 + \frac{K^2}{m_1 m_2} = 0$$

$$\lambda^2 = \frac{-\left(\frac{K}{m_1} + \frac{K}{m_2}\right) \pm \sqrt{\left(\frac{K}{m_1} + \frac{K}{m_2}\right)^2 - 4 \cdot \frac{K^2}{m_1 m_2}}}{2}$$

4 F.T

$$G = [C(sI - A)^{-1}B + D]$$

Pelo matlab calcula-se

$$G_{11} = \frac{m_1 m_2 \cdot s(m_2 \cdot s^2 + 2K)}{F}$$

$$G_{12} = \frac{K m_1 m_2 s}{F}$$

$$G_{21} = \frac{K m_1 m_2 s^2}{F}$$

$$G_{22} = \frac{-(K^2 m_2 - K^2 m_1 + K m_1 m_2 s^2)}{F}$$

onde, $F = 2K^2 m_2 - 2 m_1 K^2 + K m_2^2 \cdot s^2 + 2 m_1 \cdot K m_2 s^2 + m_1 m_2^2 \cdot s^4$

Os resultados dos auto valores e polos ficaram iguais como se nota a seguir com o programa em matlab

Código do programa

```
syms k m1 m2 s;

A=[0 0 1 0; 0 0 0 1; -k/m1 k/m2 0 0; k/m2 -k/m2 0 0];
B=[0 0; 0 0; 1/m1 0;0 1/m2];
C=[1 0 0 0; 0 0 0 1];

G=C*inv(s*eye(4,4)-A)

Polos=solve(2*k^2*m2 - 2*m1*k^2 + k*m2^2*s^2 + 2*m1*k*m2*s^2 + m1*m2^2*s^4==0)

autovalores=eig(A)
```

Resultados

```
Polos =

 -(-(2*k*m1 + k*m2 + k*(12*m1^2 - 4*m1*m2 + m2^2)^(1/2))/(2*m1*m2))^(1/2)
 -(-(2*k*m1 + k*m2 - k*(12*m1^2 - 4*m1*m2 + m2^2)^(1/2))/(2*m1*m2))^(1/2)
  (-(2*k*m1 + k*m2 + k*(12*m1^2 - 4*m1*m2 + m2^2)^(1/2))/(2*m1*m2))^(1/2)
  (-(2*k*m1 + k*m2 - k*(12*m1^2 - 4*m1*m2 + m2^2)^(1/2))/(2*m1*m2))^(1/2)


autovalores =

 -(-(k*m1 + k*m2 + k*(5*m1^2 - 2*m1*m2 + m2^2)^(1/2))/(2*m1*m2))^(1/2)
 -(-(k*m1 + k*m2 - k*(5*m1^2 - 2*m1*m2 + m2^2)^(1/2))/(2*m1*m2))^(1/2)
  (-(k*m1 + k*m2 + k*(5*m1^2 - 2*m1*m2 + m2^2)^(1/2))/(2*m1*m2))^(1/2)
  (-(k*m1 + k*m2 - k*(5*m1^2 - 2*m1*m2 + m2^2)^(1/2))/(2*m1*m2))^(1/2)
```