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- Exercício do dia 20/10 -

Ex.: Determinar as funções de transferência

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} \cdot u$$

Transformada de Laplace:

$$\begin{bmatrix} s \cdot X(s) - \cancel{X(0)}^{0} \\ s \cdot Y(s) - \cancel{Y(0)}^{0} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} \cdot U(s)$$

$$\begin{cases} s X(s) = Y(s) \Rightarrow X(s) = \dfrac{Y(s)}{s} & (1) \\ s Y(s) = -100 X(s) + 10 \cdot U(s) & (2) \end{cases}$$

Substituindo (1) em (2):

$$s Y(s) = -100 \cdot \frac{Y(s)}{s} + 10 \cdot U(s)$$

$$\Rightarrow \left(s + \frac{100}{s}\right) Y(s) = 10\, U(s) \Rightarrow \left(\frac{s^2 + 100}{s}\right) Y(s) = 10\, U(s)$$

$$\Rightarrow Y(s) = \frac{10\, s \cdot U(s)}{s^2 + 100} \quad \therefore \quad X(s) = \frac{10\, U(s)}{s^2 + 100}$$

Função de transferência:

$$F.T(x) = \frac{10}{s^2+100} \qquad F.T(y) = \frac{10s}{s^2+100}$$

1.2 -
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot u$$

↳ Por Domínio do tempo:

Solução homogênea

$$\begin{cases} \dot{x} = -x + 4y \\ \dot{y} = 5x + 2y \\ \dot{z} = -x - 3z \end{cases}$$

$$\Rightarrow \begin{bmatrix} A \cdot \lambda e^{\lambda t} \\ B \cdot \lambda e^{\lambda t} \\ C \cdot \lambda e^{\lambda t} \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} A \cdot e^{\lambda t} \\ B \cdot e^{\lambda t} \\ C \cdot e^{\lambda t} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1-\lambda & 4 & 0 \\ 5 & 2-\lambda & 0 \\ -1 & 0 & -3-\lambda \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} \cdot e^{\lambda t} = 0$$

$$\det \begin{vmatrix} -1-\lambda & 4 & 0 \\ 5 & 2-\lambda & 0 \\ -1 & 0 & -3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)(-3-\lambda)(-1-\lambda) - 5(-3-\lambda)4 - (2-\lambda)\cancel{(-1)} \cdot 0 = 0$$

$$\Rightarrow (-3-\lambda)\left[(2-\lambda)(-1-\lambda) - 20\right] = 0$$

$$\therefore (-3-\lambda) = 0 \Rightarrow \lambda_1 = -3$$

$$\therefore (2-\lambda)(-1-\lambda) - 20 = 0 \Rightarrow -2 - 2\lambda + \lambda + \lambda^2 = 20$$

$$\Rightarrow \lambda^2 - \lambda - 22 = 0 \Rightarrow \lambda = \frac{1 \pm \sqrt{1+88}}{2} = \frac{1 \pm \sqrt{89}}{2}$$

$$\therefore \quad \lambda_2 = \frac{1-\sqrt{89}}{2} \qquad \lambda_3 = \frac{1+\sqrt{89}}{2}$$

↳ Por Domínio da Frequência:

Transformada de Laplace:

$$\begin{bmatrix} s X(s) - \cancel{X(0)}^{\,0} \\ s Y(s) - \cancel{Y(0)}^{\,0} \\ s Z(s) - \cancel{Z(0)}^{\,0} \end{bmatrix} = \begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} X(s) \\ Y(s) \\ Z(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot U(s)$$

$$\begin{cases} s X(s) = -X(s) + 4Y(s) & (1) \\ s Y(s) = 5X(s) + 2Y(s) + U(s) & (2) \\ s Z(s) = -X(s) - 3Z(s) & (3) \end{cases}$$

De (1):

$$(s+1) X(s) = 4Y(s)$$

$$\Rightarrow X(s) = \frac{4Y(s)}{s+1} \quad (4)$$

Substituindo em (2):

$$s Y(s) = \frac{20 Y(s)}{s+1} + 2Y(s) + U(s)$$

$$\Rightarrow \left(s - \frac{20}{s+1} - 2\right) Y(s) = U(s)$$

$$\Rightarrow \left(\frac{s^2 + s - 20 - 2s - 2}{s+1}\right) Y(s) = U(s)$$

$$\Rightarrow Y(s) = \frac{(s+1) \cdot U(s)}{s^2 - s - 22} \quad (5)$$

Substituindo em (4):

$$X(s) = \frac{4U(s)}{s^2 - s - 22} \quad (6)$$

Substituindo em (3):

$$s Z(s) = -\frac{4U(s)}{s^2 - s - 22} - 3Z(s)$$

$$\Rightarrow (s+3) Z(s) = -\frac{4U(s)}{s^2 - s - 22} \Rightarrow Z(s) = -\frac{4U(s)}{(s^2 - s - 22)(s+3)}$$

Função de Transferência: $FT(s) = \begin{bmatrix} \dfrac{4}{s^2-s-22} \\ \dfrac{s+1}{s^2-s-22} \\ -\dfrac{4}{(s^2-s-22)(s+3)} \end{bmatrix}$

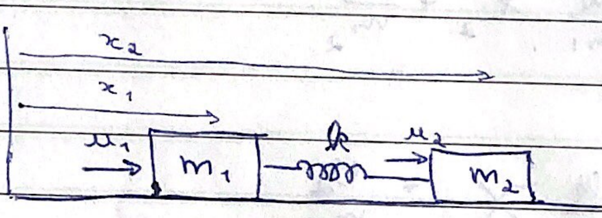
Polos:

$$(s^2 - s - 22)(s+3) = 0$$

$$s_1 = -3 \, , \quad s_2 = \frac{1-\sqrt{89}}{2} \, , \quad s_3 = \frac{1+\sqrt{89}}{2}$$

Percebe-se que os autovalores e os polos coincidem.

2-



$$m_1 \ddot{x}_1 = u_1 + k(x_2 - x_1)$$
$$m_2 \ddot{x}_2 = u_2 - k(x_2 - x_1)$$

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{m_1} & 0 & \frac{k}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_2} & 0 & -\frac{k}{m_2} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

↳ Por Domínio do tempo:

$$\det \begin{vmatrix} -\lambda & 1 & 0 & 0 \\ -\frac{k}{m_1} & -\lambda & \frac{k}{m_1} & 0 \\ 0 & 0 & -\lambda & 1 \\ \frac{k}{m_2} & 0 & -\frac{k}{m_2} & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^4 + \cancel{\frac{k^2}{m_1+m_2}} - \cancel{\frac{k^2}{m_1+m_2}} = 0$$

$$\Rightarrow \lambda^4 = 0 \Rightarrow \lambda = 0$$

↳ Por domínio da Frequência:

$$\begin{bmatrix} sX_1 - \cancel{x_1(0)}^{\,0} \\ s\dot{X}_1 - \cancel{\dot{x}_1(0)}^{\,0} \\ sX_2 - \cancel{x_2(0)}^{\,0} \\ s\dot{X}_2 - \cancel{\dot{x}_2(0)}^{\,0} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{m_1} & 0 & \frac{k}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_2} & 0 & -\frac{k}{m_2} & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ \dot{X}_1 \\ X_2 \\ \dot{X}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{U_1}{m_1} \\ 0 \\ \frac{U_2}{m_2} \end{bmatrix}$$

$$\begin{cases} sX_1 = \dot{X}_1 \\ s\dot{X}_1 = -\frac{k}{m_1}X_1 + \frac{k}{m_1}X_2 + \frac{U_1}{m_2} \\ sX_2 = \dot{X}_2 \\ s\dot{X}_2 = \frac{k}{m_2}X_1 - \frac{k}{m_2}X_2 + \frac{U_2}{m_1} \end{cases}$$

$$X_1 = \frac{-\frac{k}{m_2}\cdot\frac{u_1}{m_1} - s^2\cdot\frac{u_1}{m_1} - \frac{k}{m_1}\cdot\frac{u_2}{m_2}}{\left(\frac{k}{m_1} + \frac{k}{m_2} + s^2\right)s^2}$$

$$\dot{X}_1 = \frac{-\frac{k}{m_2}\cdot\frac{u_1}{m_1} - s^2\cdot\frac{u_1}{m_1} - \frac{k}{m_1}\cdot\frac{u_2}{m_2}}{\left(\frac{k}{m_1} + \frac{k}{m_2} + s^2\right)s}$$

$$X_2 = \frac{-\frac{k}{m_2}\cdot\frac{u_1}{m_1} - \frac{k}{m_1}\cdot\frac{u_2}{m_2} - s^2\cdot\frac{u_2}{m_2}}{\left(\frac{k}{m_1} + \frac{k}{m_2} + s^2\right)s^2}$$

$$\dot{X}_2 = \frac{-\frac{k}{m_2}\cdot\frac{u_1}{m_1} - \frac{k}{m_1}\cdot\frac{u_2}{m_2} - s^2\cdot\frac{u_2}{m_2}}{\left(\frac{k}{m_1} + \frac{k}{m_2} + s^2\right)s}$$

Polos: $s_1 = 0$, $s_2 = 0$, $s_3 = \sqrt{\frac{k}{m_1} + \frac{k}{m_2}}\, i$; $s_4 = -\sqrt{\frac{k}{m_1} + \frac{k}{m_2}}\, i$

2.2 - O resultado é o mesmo do exercício anterior. Isso porque somente se realiza uma mudança de variável ao se usar o baricentro e a distância dos blocos como variáveis de estado.

$x_G = \frac{x_1 + x_2}{2}$ | $\delta = x_2 - x_1$