Ex 22/10 - Henrique Kuhlmann - 10772672

1) $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u \rightarrow \begin{bmatrix} sX(s) \\ sY(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U(s)$

$sX(s) = Y(s)$

$sY(s) = -100\,X(s) + 10\,U(s)$

$\Rightarrow Y(s) = \dfrac{10s \cdot U(s)}{s^2+100}$ $\quad X(s) = \dfrac{10\,U(s)}{s^2+100}$

$F_{ty}(s) = \dfrac{10s}{s^2+100}$ $\quad F_{tx}(s) = \dfrac{10}{s^2+100}$

$$1.2)\quad \begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} = \underbrace{\begin{bmatrix}-1 & 4 & 0\\ 5 & 2 & 0\\ -1 & 0 & -3\end{bmatrix}}_{A}\begin{bmatrix}x\\ y\\ z\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ 1\\ 0\end{bmatrix}}_{B} u$$

Solução usual: Autovalores de A: $\lambda_1 = -3$, $\lambda_2 = \frac{1+\sqrt{89}}{2}$, $\lambda_3 = \frac{1-\sqrt{89}}{2}$

$x(t) = \alpha_1 e^{\lambda_1 t} \quad y(t) = \alpha_2 e^{\lambda_2 t} \quad z(t) = \alpha_3 e^{\lambda_3 t}$

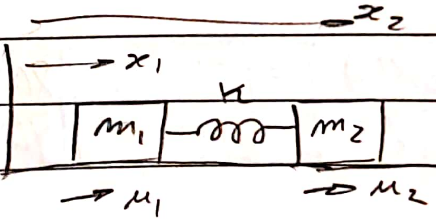
Transformada de Laplace:

$$\begin{aligned} sX(s) &= -X(s) + 4Y(s)\\ sY(s) &= 5X(s) + 2Y(s) + U(s)\\ sZ(s) &= -X(s) - 3Z(s)\end{aligned} \Rightarrow X(s) = \frac{4U}{s^2-s-22} \quad Y(s) = \frac{(1+s)U}{s^2-s-22}$$

$$Z(s) = \frac{-4U}{(s+3)(s^2-s-22)}$$

Polos: $s=3$, $s = \frac{1\pm\sqrt{89}}{2}$ (os mesmos dos autovalores)

2)



$m_2\ddot{x}_2 = u_2 - k(x_2 - x_1)$

$m_1\ddot{x}_1 = u_1 - k(x_1 - x_2)$

$$\begin{bmatrix}\dot{x}_1\\ \ddot{x}_1\\ \dot{x}_2\\ \ddot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ -k/m_1 & 0 & k/m_1 & 0\\ 0 & 0 & 0 & 1\\ k/m_2 & 0 & -k/m_2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ \dot{x}_1\\ x_2\\ \dot{x}_2\end{bmatrix} + \begin{bmatrix}0 & 0\\ 1/m_1 & 0\\ 0 & 0\\ 0 & 1/m_2\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

$\lambda^4 + (k/m_1 + k/m_2)\lambda^2 = 0 \rightarrow \lambda_1 = \lambda_2 = 0$

$\lambda_3 = i\sqrt{k/m_1 + k/m_2}$

$\lambda_4 = -i\sqrt{k/m_1 + k/m_2}$

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Transformada de Laplace:

$$X_1(s) = \frac{-\frac{K}{m_2} \cdot \frac{U_1}{m_1} - s^2 \cdot \frac{U_1}{m_1} - \frac{K}{m_1} \cdot \frac{U_2}{m_2}}{s^2 \cdot \left(\frac{K}{m_1} + \frac{K}{m_2} + s^2 \right)}$$

$$X_2(s) = \frac{-\frac{K}{m_2} \cdot \frac{U_1}{m_1} - \frac{K}{m_1} \cdot \frac{U_2}{m_2} - s^2 \cdot \frac{U_2}{m_2}}{s^2 \cdot \left(\frac{K}{m_1} + \frac{K}{m_2} + s^2 \right)}$$

$$\dot{X}_1(s) = \underline{X_1(s)} \cdot s$$

$$\dot{X}_2(s) = X_2(s) \cdot s$$

2.2) A resposta é a mesma, pois trata-se de uma mudança no sistema de coordenadas