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* Exercício aula 20/10/2020

1.1. $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U$

Transformada de Laplace:

$$\begin{bmatrix} sX(s) - X(0) \\ sY(s) - Y(0) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & 0 \end{bmatrix} \begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} U(s)$$

$$\begin{cases} sX(s) = Y(s) \\ sY(s) = -100X(s) + 10U(s) \end{cases} \Rightarrow \begin{cases} X(s) = \dfrac{10\,U(s)}{s^2+100} \\ Y(s) = \dfrac{10s\,U(s)}{s^2+100} \end{cases}$$

$$FT(y) = \frac{Y(s)}{U(s)} = \frac{10s}{s^2+100} \quad // \qquad FT(x) = \frac{X(s)}{U(s)} = \frac{10}{s^2+100} \quad //$$

1.2. $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \underbrace{\begin{bmatrix} -1 & 4 & 0 \\ 5 & 2 & 0 \\ -1 & 0 & -3 \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \\ 3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} U$

$$\det(A - \lambda I) = \left| \begin{bmatrix} -1-\lambda & 4 & 0 \\ 5 & 2-\lambda & 0 \\ -1 & 0 & -3-\lambda \end{bmatrix} \right| = (-3-\lambda)[(-1-\lambda)(2-\lambda) - 20]$$

$$= (-3-s)[(-1-s)(2-s) - 20]$$

$$s_1 = -3 \; ; \quad s_2 = -4{,}217 \quad s_3 = 5{,}217$$

$$FT(y) = \frac{s+1}{(s-2)(s+1)-20} \quad // \qquad FT(x) = \frac{1}{(s-2)(s+1)-20} \quad //$$

2.1. $m_1 \ddot{x}_1 - K(x_1 - x_2) = U_1$

$m_2 \ddot{x}_2 - K(x_2 - x_1) = U_2$

Espaço de estados:

$x_1 \quad \dot{x}_1 = x_2$

$x_2 \quad \dot{x}_2 = x_4$

$x_3 \quad \dot{x}_3 = [U_1 + K(x_1 - x_2)]/m_1$

$x_4 \quad \dot{x}_4 = [U_2 + K(x_2 - x_1)]/m_2$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ K/m_1 & -K/m_1 & 0 & 0 \\ -K/m_2 & K/m_2 & 0 & 0 \end{bmatrix} \rightarrow \det(A - \lambda I) = \left| \begin{bmatrix} -\lambda & 0 & 1 & 0 \\ 0 & -\lambda & 0 & 1 \\ K/m_1 & -K/m_1 & -\lambda & 0 \\ -K/m_2 & K/m_2 & 0 & 0 \end{bmatrix} \right|$$

$$\lambda^4 + \frac{K^2}{m_1 m_2} - \lambda^2 \left(\frac{K}{m_2} + \frac{K}{m_1} \right) = 0 \rightarrow \lambda_1 = \sqrt{\frac{K}{m}} \; ; \; \lambda_2 = -\sqrt{\frac{K}{m_2}} \; ; \; \lambda_3 = \sqrt{\frac{K}{m_1}}$$

$$\lambda_4 = \sqrt{\frac{K}{m_1}}$$