Exercícios 20/10/2020

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PME 3380

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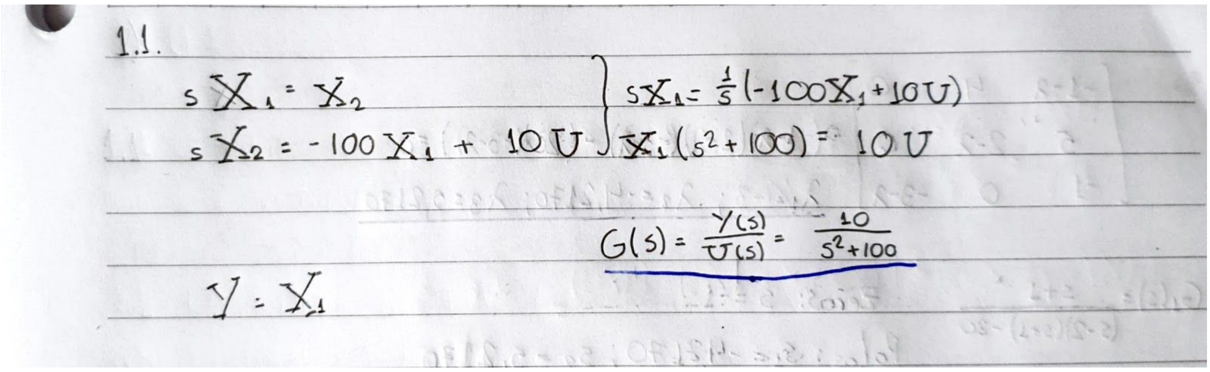
06/09/2020

Exercícios para casa:

1.
    1.1.



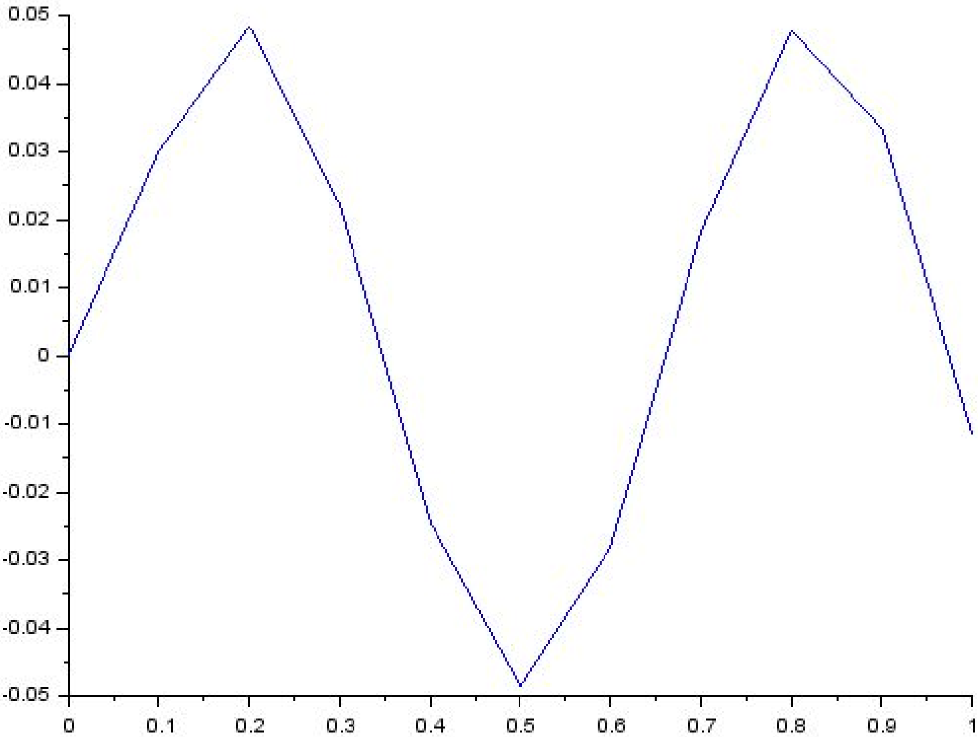
1.1.

$$\left.\begin{aligned} sX_1 &= X_2 \\ sX_2 &= -100X_1 + 10U \end{aligned}\right\} \begin{aligned} sX_1 &= \frac{1}{s}(-100X_1 + 10U) \\ X_1(s^2+100) &= 10U \end{aligned}$$

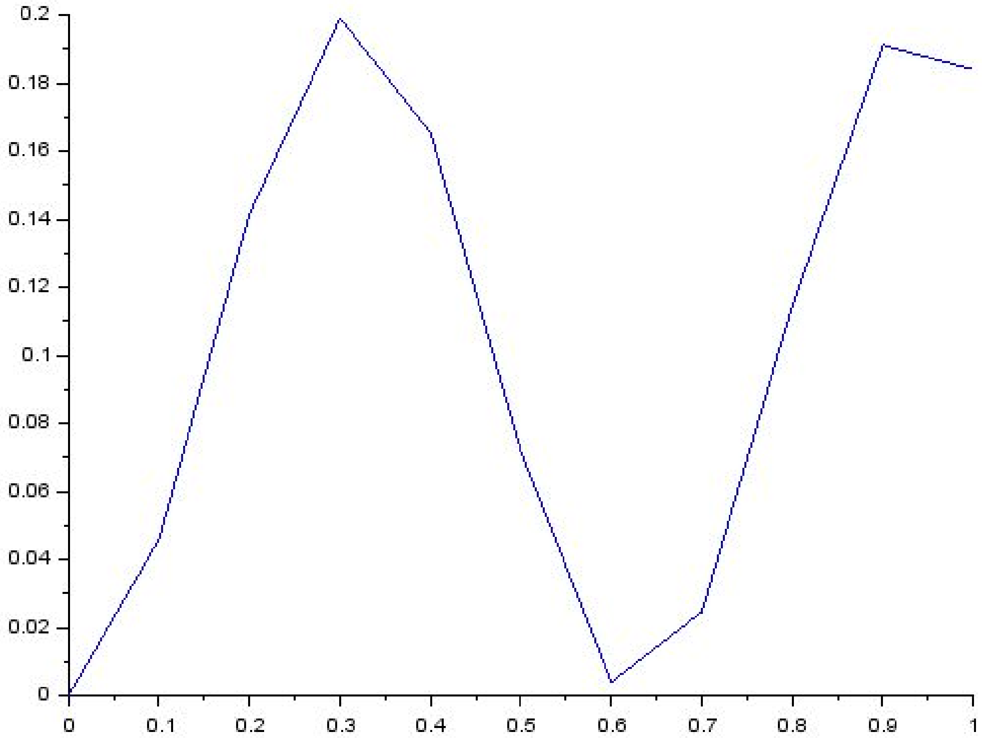
$$G(s) = \frac{Y(s)}{U(s)} = \frac{10}{s^2+100}$$

$$Y = X_1$$

Impulso:



Degrau:


0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

1.2.

1.2

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}-1 & 4 & 0\\ 5 & 2 & 0\\ -1 & 0 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 1\\ 0\end{bmatrix} u \Rightarrow \begin{array}{l} sX_1 = -1X_1 + 4X_2 \\ sX_2 = 5X_1 + 2X_2 + U \\ sX_3 = -X_1 - 3X_3 \end{array}$$

$$\begin{matrix}Y_1\\ Y_2\end{matrix} = \begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \Rightarrow \begin{array}{l} Y_1 = X_2 \\ Y_2 = X_1 \end{array}$$

$$sX_1 = -X_1 + 4(5X_1 + U)/s-2$$

$$((s+1)(s-2) - 20)X_1 = 4U$$

$$G_2(s) = \frac{Y_2(s)}{U(s)} = \frac{4}{(s+1)(s-2)-20}$$

$$sX_2 = 2X_2 + 5\left(\frac{4}{s+1}X_2\right) + U$$

$$\left(s-2 - \frac{20}{s+1}\right)X_2 = U$$

$$G_1(s) = \frac{Y_1(s)}{U(s)} = \frac{(s+1)\cdot 1}{(s-2)(s+1)-20}$$

$$\left.\begin{array}{l} \dot{x}_1 = -1x_1 + 4x_2 \\ \dot{x}_2 = 5x_1 + 2x_2 + 1u \\ \dot{x}_3 = -1x_1 - 3x_3 \end{array}\right\} \dot{x}_1 = \dot{x}_3 + 3x_3 + 2(\dot{x}_2 - 5x_1 - u)$$

$$\begin{bmatrix}-1-\lambda & 4 & 0\\ 5 & 2-\lambda & 0\\ -1 & 0 & -3-\lambda\end{bmatrix} \Rightarrow (-1-\lambda)(2-\lambda)(-3-\lambda) - (20(-3-\lambda)) = 0$$

$\lambda_1 = -3$; $\lambda_2 = -4{,}2170$; $\lambda_3 = 5{,}2170$

$G_1(s) = \frac{s+1}{(s-2)(s+1)-20}$ Zeros: $s = -1$

Polos: $s_1 = -4{,}2170$; $s_2 = 5{,}2170$

$G_2(s) = \frac{1}{(s-2)(s+1)-20}$ Zeros: $s =$ Indefinido

Polos: $s_1 = -4{,}2170$; $s_2 = 5{,}2170$

Os valores encontrados para os Polos são os mesmos que os do autovalor para matriz A.

2.

2.1.

2.1

$$x_1 = x_1 \qquad \dot{x}_1 = x_3$$

$$m_1\ddot{x}_1 - K(x_1 - x_2) = U_1 \qquad x_2 = x_2 \qquad \dot{x}_2 = x_4$$

$$m_2\ddot{x}_2 - K(x_2 - x_1) = U_2 \qquad x_3 = \dot{x}_1 \qquad \dot{x}_3 = [U_1 + K(x_1 - x_2)]/m_1$$

$$x_4 = \dot{x}_2 \qquad \dot{x}_4 = [U_2 + K(x_2 - x_1)]/m_2$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K}{m_1} & \frac{-K}{m_1} & 0 & 0 \\ \frac{-K}{m_2} & \frac{K}{m_2} & 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix}; \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad D = 0$$

Autovalores: $\lambda^4 + \frac{K^2}{m_1 m_2} - \left(\lambda^2 \cdot \frac{K}{m_2}\right) - \left(\lambda^2 \frac{K}{m_1}\right) = 0$

$$\lambda_1 = \sqrt{\frac{K}{m_2}}, \quad \lambda_2 = -\sqrt{\frac{K}{m_2}}, \quad \lambda_3 = \sqrt{\frac{K}{m_1}}, \quad \lambda_4 = \sqrt{\frac{K}{m_1}}$$