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1) $\mathcal{L}[Af(t)] = \int_0^\infty Af(t)e^{-st}dt = A\cdot\int_0^\infty f(t)e^{-st}dt = AF(s)$ ($F(t)$)

2) $\mathcal{L}[f_1(t) \pm f_2(t)] = \int_0^\infty (f_1(t) \pm f_2(t))e^{-st}dt = \int_0^\infty f_1(t)e^{-st}dt \pm \int_0^\infty f_2(t)e^{-st}dt =$

$= F_1(s) \pm F_2(s)$

3) $\int_0^t f(t)dt = g(t) - g(0)$; $g(t) = f(t)$

$\mathcal{L}[g(t) - g(0)] = \int_0^\infty (g(t) - g(0))e^{-st}dt = -(g(t) - g(0))\frac{e^{-st}}{s}\Big|_0^\infty + \int_0^\infty f(t)e^{-st}dt$

$= \frac{1}{s}\int_0^\infty f(t)e^{-st}dt = \frac{F(s)}{s}$ //

4) $\mathcal{L}[e^{\alpha t}f(t)] = \int_0^\infty f(t)e^{-\alpha t}e^{-st}dt = \int_0^\infty f(t)\cdot e^{-(\alpha+s)t}dt = \int_0^\infty f(t)e^{-kt}dt = F(k)$

$= F(\alpha + s)$

5) $\mathcal{L}[f(\frac{t}{a})] = \int_0^\infty f(\frac{t}{a})e^{-st}dt = \int_0^\infty f(\theta)e^{-as\theta}a\,d\theta = a\int_0^\infty f(\theta)e^{-as\theta}d\theta$

$= aF(as)$ //

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6) $\mathcal{L}[f_1 * f_2] = \int_0^\infty \int_0^t f_1(z)\cdot f_2(t-z)\,dz\,e^{-st}dt = \int_0^\infty \int_0^\infty f_1(z)\,f_2(t-z)dt\,e^{-st}$

$= \int_0^\infty f_1(z)\int_z^\infty f_2(t-z)dt\,e^{-st}dz$

$t - z = v \Rightarrow dt = dv \Rightarrow \int_0^\infty f_1(z)\int f_2(v)dv\,e^{-s(v+z)}dz =$

$\int f_1(z)e^{-sz}dz \int f_2(v)e^{-sv}dv = F_1(s)\cdot F_2(s) = \mathcal{L}[f_1 * f_2]$ //