Escola Politécnica da USP

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PME 3380 - Modelagem de Sistemas Dinâmicos - Ex Aula 15/10

1) $L\{A f(t)\} = A\int_0^{\infty} F(\tau) \cdot e^{-s\tau} d\tau = A F(s)$

2) $L\{f_1(\tau) \pm f_2(\tau)\} = \int_0^{\infty} (f_1(\tau) + f_2(\tau))\, e^{-s\tau} d\tau = \int_0^{+\infty} f_1(\tau)\, e^{-s\tau} d\tau \pm \int_0^{\infty} f_1(\tau)\, e^{-s\tau} d$

Portanto: $\underline{L\{f_1(\tau) \pm f_2(\tau)\} = F_1(s) + F_2(s)}$

3) $L\{\int f(\tau)d\tau\} = \int_0^{\infty} \left(\int f(\tau)d\tau\right) e^{-s\tau} d\tau = -\frac{e^{-s\tau}}{s} \int f(\tau)d\tau \Big|_0^{\infty} + \int_0^{\infty} \frac{e^{-s\tau}}{s} f(t) d\tau$

$\therefore \boxed{L\{\int f(t) d\tau\} = \frac{1}{s}\int f(\tau)d\tau + \frac{F(s)}{s}}$

4) $L\{e^{-a\tau} F(\tau)\} = \int_0^{\infty} e^{-a\tau} F(\tau) e^{-s\tau} d\tau = F(s+a)$

5) $L\{F(\frac{\tau}{a})\} = \int_0^{\infty} f(\frac{\tau}{a})\, e^{-s\tau} d\tau = \int_0^{\frac{\infty}{a}} F(u)\, e^{-sua} du \cdot a = a\int_0^{\infty} f(u)\, e^{-sua} du$

6)

Da propriedade $L\left(\frac{dF}{d\tau}\right) = sF(s) - F(0)$

$\lim_{s\to 0} L\left(\frac{dF}{d\tau}\right) = \lim_{s\to 0} (sF(s) - F(0))$

Daí: $\lim_{s\to 0} \int_0^{\infty} \frac{dF}{d\tau} \lim_{s\to 0} e^{-s\tau} d\tau = -F(0) + \lim_{s\to 0} sF(s) =$

$\Rightarrow F(\tau)\Big|_0^{\infty} = -\cancel{F(0)} + \lim_{s\to 0} sF(s) = F(\infty) - \cancel{F(0)}$

$\therefore \boxed{\lim F(s) = \lim F(\tau)}$

7. Sendo $f(t) = e^{-at}$

$$L\{f(t)\} = L\{e^{-at}\} = \int_0^\infty e^{-at} e^{-st}\, dt = \int_0^\infty e^{-t(a+s)}\, dt$$

$$\Rightarrow L\{f(t)\} = \frac{e^{-(a+s)t}}{-(a+s)}\Big|_0^\infty = \lim_{t\to\infty} \cancel{\frac{e^{-(a+s)t}}{-(a+s)}} - \frac{e^{-(a+s)\cdot 0}}{-(a+s)}$$

$$\Rightarrow \boxed{L\{f(t)\} = \frac{1}{a+s}}$$ //

8. Sendo $f(t) = t e^{-at}$

$$L\{f(t)\} = L\{t e^{-at}\} = \int_0^\infty t e^{-at} e^{-st}\, dt = \int_0^\infty t e^{(-a+s)t}\, dt$$

$$\boxed{L\{f(t)\} = 0 - \int_0^\infty \frac{e^{-(a+s)t}}{-(a+s)}\, dt = \frac{1}{(s+a)^2}}$$ //

9. $\bullet\ L\{\text{sen}(\omega t)\} = \int_0^\infty \text{sen}(\omega t) e^{-st}\, dt = \text{sen}\,\omega t\left(-\frac{1}{s}e^{-st}\right)\Big|_0^\infty + \frac{\omega}{s}\int_0^\infty \cos(\omega t)\, e^{-st}\, dt$

$$L\{\text{sen}(\omega t)\} = \frac{\omega}{s} L\{\cos(\omega t)\} \Rightarrow L\{\cos \omega t\} = \frac{s}{\omega} L\{\text{sen}(\omega t)\}$$

$$\Rightarrow L\{\cos(\omega t)\} = \int_0^\infty \cos \omega t\, e^{-st}\, dt = \cos \omega t\left(-\frac{1}{s}e^{-st}\right)\Big|_0^\infty + \frac{\omega}{s}\int_0^\infty \text{sen}\,\omega t\, e^{-st}\, dt$$

$$\therefore L\{\cos \omega t\} = \frac{1}{s} - \frac{\omega}{s} L\{\text{sen}\,\omega t\}$$

Daí $\Rightarrow \frac{s}{\omega} L\{\text{sen}\,\omega t\} = \frac{1}{s} - \frac{\omega}{s} L\{\text{sen}\,\omega t\} \Rightarrow \boxed{L\{\text{sen}\,\omega t\} = \frac{\omega}{s^2+\omega^2}}$

$$\downarrow$$

$$\boxed{L\{\cos \omega t\} = \frac{s}{s^2+\omega^2}}$$

10. $L\{f*g\} = \int_0^\infty (f*g)\, e^{-st}\, dt = \int_0^\infty e^{-st} \int_0^\infty f(t-\sigma) g(\sigma)\, d\sigma)\, dt$

$$= \int_0^\infty \int_0^\infty f(t-\sigma) g(\sigma) e^{-st}\, d\sigma\, dt$$

$\theta = t - \sigma \Rightarrow d\theta = dt$

$$\Rightarrow L\{f*g\} = \int_0^\infty \int_0^\infty f(\theta) g(\sigma) \cdot e^{-s(\theta+\sigma)}\, d\theta\, d\sigma = \int_0^\infty \ldots \quad \therefore \boxed{L\{f*g\} = F(s)G(s)}$$

$$= \underbrace{\int_0^\infty f(\theta) e^{-s\theta}\, d\theta}_{F(s)} \cdot \underbrace{\int_0^\infty g(\sigma) e^{-s\sigma}\, d\sigma}_{G(s)}$$