Exercicios p/ 29/10

1) $\mathcal{L}[A f(t)] = \int_0^\infty A f(t) e^{-st} dt = A F(s)$

2) $\mathcal{L}[f_1(t) \pm f_2(t)] = \int_0^\infty (f_1(t) \pm f_2(t)) e^{-st} dt = \int_0^\infty f_1(t) e^{-st} dt \pm \int_0^\infty f_2(t) e^{-st} dt = F_1(s) \pm F_2(s)$

3) Com $f(t) = \frac{dg(t)}{dt}$

$\mathcal{L}[\int f(t) dt] = \int_0^\infty g(t) e^{-st} dt \Rightarrow$ Integrando: $\int_0^\infty g(t) e^{-st} dt = -\frac{g(t) e^{-st}}{s}\Big|_0^\infty - \int_0^\infty \frac{dg(t)}{dt}\left(-\frac{e^{-st}}{s}\right) dt =$

$= \frac{g(0)}{s} + \frac{1}{s}\int_0^\infty f(t) e^{-st} dt = \frac{[\int f(t) dt]_{t=0} + F(s)}{s}$

4) $\mathcal{L}[e^{-at} f(t)] = \int_0^\infty e^{-at} f(t) e^{-st} dt = \int_0^\infty e^{-(s+a)t} f(t) dt = F(s+a)$

5) $\mathcal{L}[f(t/a)] = \int_0^\infty f(t/a) e^{-st} dt = \int_0^\infty f(t/a) e^{-sa \cdot t/a} a\, d(t/a) = a F(sa)$

6)

$\lim_{s \to \infty} \int_{0^+}^\infty \frac{df(t)}{dt} e^{-st} dt = \int_{0^+}^\infty \frac{df(t)}{dt} \lim_{s \to \infty} e^{-st} dt = 0$

$\lim_{s \to \infty} [sF(s) - f(0^+)] = 0 \Rightarrow \lim_{s \to \infty} f(0^+) = \lim_{t \to 0^+} f(t) \Rightarrow \lim_{s \to \infty} sFs - \lim_{t \to 0^+} f(t) = 0$

7) $\mathcal{L}[e^{-at}] = \int_0^\infty e^{-at} e^{-st} dt = -\frac{e^{-(s+a)t}}{s+a}\Big|_0^\infty = 0 - \left(-\frac{1}{s+a}\right) = \frac{1}{s+a}$

8) $\mathcal{L}[te^{-at}] = \int_0^\infty t e^{-(s+a)t} dt = \frac{t e^{-(s+a)t}}{s+a}\Big|_0^\infty - \int_0^\infty -\frac{e^{-(s+a)t}}{s+a} dt = 0 + \left(\frac{1}{s+a} \cdot \frac{1}{s+a}\right) = \frac{1}{(s+a)^2}$

9) $\mathcal{L}[\operatorname{sen}\omega t] = \int_0^\infty \operatorname{sen}\omega t\, e^{-st}\,dt = -\frac{e^{-st}}{s}\operatorname{sen}\omega t \Big|_0^\infty - \int_0^\infty -\frac{e^{-st}}{s}\omega\cos\omega t\,dt = \int_0^\infty \frac{e^{-st}}{s}\omega\cos\omega t\,dt$

$= -\frac{e^{-st}}{s^2}\omega\cos\omega t - \int_0^\infty \frac{e^{-st}}{s^2}\omega^2\operatorname{sen}\omega t\,dt = \frac{\omega}{s^2} - \frac{\omega^2}{s^2}\int_0^\infty e^{-st}\operatorname{sen}\omega t\,dt \Rightarrow \int_0^\infty e^{-st}\operatorname{sen}\omega t\,dt$

$\Rightarrow \frac{\omega}{s^2} = \int_0^\infty e^{-st}\operatorname{sen}\omega t\,dt + \frac{\omega^2}{s^2}\int_0^\infty e^{-st}\operatorname{sen}\omega t\,dt \Rightarrow \int_0^\infty e^{-st}\operatorname{sen}\omega t\,dt = \frac{\frac{\omega}{s^2}}{\frac{(s^2+\omega^2)}{s^2}} = \underline{\frac{\omega}{s^2+\omega^2}}$ //

10) $\mathcal{L}[f\cdot g] = \int_0^\infty (f\cdot g)e^{-st}dt = \int_0^\infty e^{-st}\left(\int_0^\infty f(t-\alpha)\,g(t-\alpha)\,d\alpha\right)dt = F(s)\cdot G(s)$

11) $y(t) = \frac{7}{3}e^{-t} + \frac{3}{2}e^{-2t} - \frac{1}{6}e^{-4t} \Rightarrow Y(s) = \frac{7}{3(s+1)} + \frac{3}{2(s+2)} - \frac{1}{6(s+4)}$

$U(s) = 1 \Rightarrow G(s) = \frac{Y(s)}{U(s)} = Y(s) = \frac{7}{3s+3} + \frac{3}{2s+4} - \frac{1}{6s+24}$

12) $y(t) = 1 - \frac{7}{3}e^{-t} + \frac{3}{2}e^{-2t} - \frac{1}{6}e^{-4t} \Rightarrow Y(s) = \frac{1}{s} - \frac{7}{3(s+1)} + \frac{3}{2(s+2)} - \frac{1}{6(s+4)}$

$U(s) = 1 \Rightarrow G(s) = \frac{Y(s)}{U(s)} = Y(s) = \frac{1}{s} - \frac{7}{3s+3} + \frac{3}{2s+4} - \frac{1}{6s+24}$

13) I $L_a D i_a + R_a i_a = e_a - k_b\omega$ $\Rightarrow$ I $(L_a s + R_a)I_a = E_a - k_b s$

II $J\ddot{\theta} + B\dot{\theta} = K i_a$ II $(Js^2 + Bs) = K I_a$

$G_1(s) = \frac{Y(s)}{U(s)} = \frac{K}{JL_a s^3 + (R_a J + BL_a)s^2 + (R_a B + K\cdot K_b)s}$

Descobrimos as raízes do denominador para obter os pólos S_i