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- Exercícios do dia 15/10 -

1- $\mathcal{L}[Af(t)] = \int_0^\infty A\,f(t) \cdot e^{-st}\,dt = A\int_0^\infty f(t) \cdot e^{-st}\,dt$

$= A \cdot F(s)$ //

2- $\mathcal{L}[f_1(t) \pm f_2(t)] = \int_0^\infty [f_1(t) \pm f_2(t)] \cdot e^{-st}\,dt$

$= \int_0^\infty f_1(t) \cdot e^{-st}\,dt \pm \int_0^\infty f_2(t) \cdot e^{-st}\,dt$

$= F_1(s) \pm F_2(s)$ //

3- $\mathcal{L}\left[\int f(t)\,dt\right] = \int_0^\infty \left(\int f(t)\,dt\right) e^{-st}\,dt = *$

$u = \int f(t)\,dt$ $\qquad dv = e^{-st}\,dt$

$du = f(t)\,dt$ $\qquad v = -\frac{e^{-st}}{s}$

$* = \frac{1}{s}\int f(t)\,dt + \frac{F(s)}{s}$ //

4- $\mathcal{L}[e^{-at}f(t)] = \int_0^\infty e^{-at} \cdot f(t) \cdot e^{-st}\,dt$

$= \int_0^\infty e^{-(a+s)t} \cdot f(t)\,dt = F(a+s)$ //

5- $\mathcal{L}[f(\frac{t}{a})] = \int_0^\infty f(\frac{t}{a}) \cdot e^{-st}\,dt = *$

$u = \frac{t}{a} \Rightarrow du = \frac{dt}{a} \Rightarrow dt = a\,du$

$\hookrightarrow t = au$

$$* = \int_0^\infty f(u)\, e^{-sau} \cdot a\, du = a \int_0^\infty f(u) \cdot e^{-sau}\, du$$

$$= a \cdot F(sa)$$

6- $\lim\limits_{t \to \infty} f(t) = \lim\limits_{s \to 0} s \cdot F(s)$

Como $\mathcal{L}\left(\frac{df}{dt}\right) = s \cdot F(s) - f(0) \Rightarrow \lim\limits_{s \to 0} \mathcal{L}\left(\frac{df}{dt}\right) = \lim\limits_{s \to 0}\left(s F(s) \cdot f(0)\right)$

$$\lim_{s \to 0} \left[\int_0^\infty \frac{df}{dt} \cdot e^{-st}\, dt\right] = -F(0) + \lim_{s \to 0} s \cdot F(s)$$

$$\Rightarrow \int_0^\infty \frac{df}{dt} \left(\lim_{s \to 0} \cancel{e^{-st}}^{1}\right) dt = -F(0) + \lim_{s \to 0} s \cdot F(s)$$

$$\Rightarrow \lim_{s \to 0} s \cdot F(s) - F(0) = F(t)\Big|_0^\infty = F(\infty) - F(0)$$

$$\therefore \lim_{s \to 0} s \cdot F(s) = \lim_{t \to \infty} F(t)$$

7- $F(t) = e^{-at}$

$$\mathcal{L}\left[e^{-at}\right] = \int_0^\infty e^{-at} \cdot e^{-st}\, dt = \int_0^\infty e^{-(a+s)t}\, dt = \frac{e^{-(a+s)t}}{-(a+s)}\Bigg|_0^\infty$$

$$= \lim_{t \to \infty} \left[\frac{\cancel{e^{-(a+s)t}}^{0}}{-(a+s)}\right] - \frac{e^0}{-(a+s)} = \frac{1}{a+s}$$

8- $f(t) = t \cdot e^{-at}$

$$\mathcal{L}[t \cdot e^{-at}] = \int_0^\infty t \cdot e^{-at} \cdot e^{-st}\,dt = \int_0^\infty t \cdot e^{-(a+s)t}\,dt = *$$

$u = t \qquad dv = e^{-(a+s)t}\,dt$

$du = dt \qquad v = \dfrac{e^{-(a+s)t}}{-(a+s)}$

$$* = \cancel{t \cdot \frac{e^{-(a+s)t}}{-(a+s)}\Bigg|_0^\infty}^{\,0} - \int_0^\infty \frac{e^{-(a+s)t}}{-(a+s)}\,dt = \frac{1}{(a+s)^2}$$

a) $$\mathcal{L}[\text{sen}(wt)] = \int_0^\infty \text{sen}(wt) \cdot e^{-st}\,dt = *$$

$u = \text{sen}(wt) \qquad dv = e^{-st}\,dt$

$du = w\cos(wt) \qquad v = -\frac{1}{s} \cdot e^{-st}$

$$* = \cancel{\text{sen}(wt)\left(-\frac{1}{s} \cdot e^{-st}\right)\Bigg|_0^\infty}^{\,0} + \frac{w}{s}\int_0^\infty \cos(wt)\, e^{-st}\,dt$$

$$= \frac{w}{s} \cdot \mathcal{L}[\cos(wt)] \Rightarrow \mathcal{L}[\cos(wt)] = \frac{s}{w}\mathcal{L}[\text{sen}(wt)] \quad (1)$$

$$\mathcal{L}[\cos(wt)] = \cos(wt)\left(-\frac{1}{s} \cdot e^{-st}\right)\Bigg|_0^\infty - \frac{w}{s}\int_0^\infty \text{sen}(wt) \cdot e^{-st}\,dt$$

$$= \frac{1}{s} - \frac{w}{s}\mathcal{L}[\text{sen}(wt)] \quad (2)$$

Substituindo (1) em (2):

$$\frac{s}{w}\mathcal{L}[\text{sen}(wt)] = \frac{1}{s} - \frac{w}{s}\mathcal{L}[\text{sen}(wt)]$$

$$\left(\frac{s}{w}+\frac{w}{s}\right)\mathcal{L}[\operatorname{sen}(wt)]=\frac{1}{s}$$

$$\Rightarrow \frac{s^2+w^2}{w\cancel{s}}\cdot\mathcal{L}[\operatorname{sen}(wt)]=\frac{1}{\cancel{s}}$$

$$\therefore \mathcal{L}[\operatorname{sen}(wt)]=\frac{w}{s^2+w^2} \quad //$$

$$\mathcal{L}[\cos(wt)]=\frac{s}{s^2+w^2} \quad //$$

10- $\mathcal{L}[f*g]=\int_0^\infty (f*g)\,e^{-st}\,dt=\int_0^\infty\left(\int_0^\infty f(t-\tau)\,g(\tau)\,d\tau\right)e^{-st}\,dt$

$$=\int_0^\infty\int_0^\infty f(t-\tau)\,g(\tau)\,e^{-st}\,dt\,d\tau=*$$

$\alpha=t-\tau \Rightarrow d\alpha=dt$

$$=*\int_0^\infty\int_0^\infty f(\alpha)\,g(\tau)\cdot e^{-s(\alpha+\tau)}\,d\alpha\,d\tau$$

$$=\left(\int_0^\infty f(\alpha)\,e^{-s\alpha}\,d\alpha\right)\left(\int_0^\infty g(\tau)\,e^{-s\tau}\,d\tau\right)$$

$$=F(s)\cdot G(s) \quad //$$

11- $y(t) = \frac{7}{3} \cdot e^{-t} + \frac{3}{2} \cdot e^{-2t} - \frac{1}{6} \cdot e^{-4t}$

$\mathcal{L}[y(t)] = Y(s) = \frac{7}{3} \cdot \frac{1}{s+1} + \frac{3}{2} \cdot \frac{1}{s+2} - \frac{1}{6} \cdot \frac{1}{\cancel{s}+4}$

$u(t) = \delta(t) \Rightarrow \mathcal{L}[u(t)] = U(s) = 1$

$G(s) = \frac{Y(s)}{U(s)} = \frac{Y(s)}{1} \Rightarrow G(s) = Y(s)$

$G(s) = \frac{7}{3} \cdot \frac{1}{s+1} + \frac{3}{2} \cdot \frac{1}{s+2} - \frac{1}{6} \cdot \frac{1}{s+4}$

12- $y(t) = 1 - \frac{7}{3} \cdot e^{-t} + \frac{3}{2} \cdot e^{-2t} - \frac{1}{6} \cdot e^{-4t}$

$\mathcal{L}[y(t)] = \frac{1}{s} - \frac{7}{3} \cdot \frac{1}{s+1} + \frac{3}{2} \cdot \frac{1}{s+2} - \frac{1}{6} \cdot \frac{1}{s+4}$

Degrau de entrada:

$u(t) = 1 \Rightarrow U(s) = \frac{1}{s}$

$G(s) = \frac{Y(s)}{U(s)} \quad \therefore \quad G(s) = 1 - \frac{7}{3} \cdot \frac{s}{s+1} + \frac{3}{2} \cdot \frac{s}{s+2} - \frac{1}{6} \cdot \frac{s}{s+4}$