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* PME3361 - Lista do dia 15/10

01) $L[Af(t)] = \int_0^{+\infty} Af(t)e^{-st}dt = A\underbrace{\int_0^{+\infty} f(t)e^{-st}dt}_{\parallel} \Rightarrow \boxed{L[Af(t)] = AF(s)}$

02) $L[f_1(t)+f_2(t)] = \int_0^{+\infty}[f_1(t)+f_2(t)]e^{-st}dt = \int_0^{+\infty} f_1(t)e^{-st}dt + \int_0^{+\infty} f_2(t)e^{-st}dt$

$\Rightarrow \boxed{L[f_1(t)+f_2(t)] = F_1(s)+F_2(s)}$

$L[f_1(t)-f_2(t)] = \int_0^{+\infty}[f_1(t)-f_2(t)]e^{-st}dt = \int_0^{+\infty} f_1(t)e^{-st}dt - \int_0^{+\infty} f_2(t)e^{-st}dt$

$\Rightarrow \boxed{L[f_1(t)-f_2(t)] = F_1(s)-F_2(s)}$

03) $L\left[\int f(t)dt\right] = \int_0^{+\infty}\left[\int f(t)dt\right]e^{-st}dt \rightarrow$ por partes: $p = \int f(t)dt \Rightarrow dp = f(t)dt$

$dq = e^{-st}dt \Rightarrow q = -\frac{e^{-st}}{s}$

$\rightarrow pq\Big|_0^{+\infty} - \int_0^{+\infty} q\,dp \Rightarrow \int f(t)dt\cdot\left(-\frac{e^{-st}}{s}\right)\Bigg|_0^{+\infty} - \int_0^{+\infty} -\frac{e^{-st}}{s}f(t)dt \Rightarrow$

$\Rightarrow \int f(t)dt\left(-\cancelto{0}{\frac{e^{-st}}{s}}\right)\Bigg|_{t=+\infty} - \int f(t)dt\left(-\frac{e^{-st}}{s}\right)\Bigg|_{t=0\pm} + \frac{1}{s}\int_0^{+\infty} f(t)e^{-st}dt \Rightarrow$

$\Rightarrow \boxed{L\left[\int f(t)dt\right] = \frac{1}{s}\left[\left(\int f(t)dt\right)\Big|_{t=0\pm}\right] + \frac{F(s)}{s}}$

04) $L[e^{-at}f(t)] = \int_0^{+\infty} e^{-at}f(t)e^{-st}dt = \int_0^{+\infty} f(t)e^{-(s+a)t}dt \Rightarrow \boxed{L[e^{-at}f(t)] = F(s+a)}$

05) $L\left[f\left(\frac{t}{a}\right)\right] = \int_0^{+\infty} f\left(\frac{t}{a}\right)e^{-st}dt \rightarrow$ mudança de variável: $\frac{t}{a} = p \rightarrow dt = a\,dp$

$\Rightarrow \int_0^{+\infty} f(p)e^{-sap}a\,dp = a\int_0^{+\infty} f(p)e^{-sap}dp \Rightarrow \boxed{L\left[f\left(\frac{t}{a}\right)\right] = aF(as)}$

06) $\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s) \Rightarrow L(\dot{f}) = sF(s) - f(0) \Rightarrow \lim_{s\to\infty} L(\dot{f}) = \lim_{s\to\infty} sF(s) - f(0)$

$\Rightarrow \lim_{s\to\infty}\int_0^{+\infty}\dot{f}e^{-st}dt = -f(0) + \lim_{s\to\infty} sF(s) \Rightarrow \int_0^{+\infty}\dot{f}\cancelto{0}{\lim_{s\to\infty} e^{-st}}dt = -f(0) + \lim_{s\to\infty} sF(s)$

$\Rightarrow \lim_{s\to\infty} sF(s) = f(0) \Rightarrow \boxed{\therefore \lim_{s\to\infty} sF(s) = \lim_{t\to 0} f(t)}$ c.q.d.

07) $L[e^{-at}] = \int_0^{+\infty} e^{-at} e^{-st}\,dt = \int_0^{+\infty} e^{-(s+a)t}\,dt = -\frac{e^{-(s+a)t}}{s+a}\Big|_0^{+\infty} =$

$$= \underbrace{-\frac{e^{-(s+a)t}}{s+a}\Big|_{t=+\infty}}_{0} - \left(-\frac{e^{-(s+a)t}}{s+a}\right)\Big|_{t=0} \Rightarrow \boxed{L[e^{-at}] = \frac{1}{s+a}}$$

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08) $L[te^{-at}] = \int_0^{+\infty} te^{-at} e^{-st}\,dt = \int_0^{+\infty} te^{-(s+a)t}\,dt \rightarrow$ por partes: $p = t \Rightarrow dp = dt$

$dq = e^{-(s+a)t}\,dt$

$\hookrightarrow q = -\frac{e^{-(s+a)t}}{s+a}$

$$\hookrightarrow pq\Big|_0^{+\infty} - \int_0^{+\infty} q\,dp \Rightarrow \underbrace{t\left(-\frac{e^{-(s+a)t}}{s+a}\right)\Big|_{t=+\infty}}_{0} - \underbrace{t\left(-\frac{e^{-(s+a)t}}{s+a}\right)\Big|_{t=0}}_{0} +$$

$$-\int_0^{+\infty} -\frac{e^{-(s+a)t}}{s+a}\,dt \Rightarrow \int_0^{+\infty} \frac{e^{-(s+a)t}}{s+a}\,dt \Rightarrow \underbrace{-\frac{e^{-(s+a)t}}{(s+a)^2}\Big|_{t=+\infty}}_{0} - \left(-\frac{e^{-(s+a)t}}{(s+a)^2}\Big|_{t=0}\right)$$

$$\Rightarrow \boxed{L[te^{-at}] = \frac{1}{(s+a)^2}}$$

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09) $L[\text{sen}\,wt] = \int_0^{+\infty} \text{sen}\,wt\, e^{-st}\,dt \rightarrow$ por partes: $p = \text{sen}\,wt \rightarrow dp = w\cos wt\,dt$

$dq = e^{-st}\,dt \rightarrow q = -\frac{e^{-st}}{s}$

$$\hookrightarrow pq\Big|_0^{+\infty} - \int_0^{+\infty} q\,dp \Rightarrow \underbrace{\text{sen}\,wt\left(-\frac{e^{-st}}{s}\right)\Big|_{t=+\infty}}_{0} - \underbrace{\text{sen}\,wt\left(-\frac{e^{-st}}{s}\right)\Big|_{t=0}}_{0} - \int_0^{+\infty} -\frac{e^{-st}}{s} w\cos wt\,dt$$

$$\Rightarrow L[\text{sen}\,wt] = \frac{w}{s}\int_0^{+\infty} \cos wt\, e^{-st}\,dt = \frac{w}{s} L[\cos wt]$$

$L[\cos wt] = \int_0^{+\infty} \cos wt\, e^{-st}\,dt \rightarrow$ por partes: $p = \cos wt \rightarrow dp = -w\,\text{sen}\,wt\,dt$

$dq = e^{-st}\,dt \rightarrow q = -\frac{e^{-st}}{s}$

$$\hookrightarrow pq\Big|_0^{+\infty} - \int_0^{+\infty} q\,dp \Rightarrow \underbrace{\cos wt\left(-\frac{e^{-st}}{s}\right)\Big|_{t=+\infty}}_{0} - \cos wt\left(-\frac{e^{-st}}{s}\right)\Big|_{t=0} - \int_0^{+\infty} -\frac{e^{-st}}{s}(-w\,\text{sen}\,wt)\,dt$$

$$\Rightarrow L[\cos wt] = \frac{1}{s} - \frac{w}{s}\int_0^{+\infty} \text{sen}\,wt\, e^{-st}\,dt = \frac{1}{s} - \frac{w}{s} L[\text{sen}\,wt]$$

$$\hookrightarrow L[\text{sen}\,wt] = \frac{w}{s}\left[\frac{1}{s} - \frac{w}{s} L[\text{sen}\,wt]\right] \Rightarrow L[\text{sen}\,wt] + \frac{w^2}{s^2} L[\text{sen}\,wt] = \frac{w}{s^2}$$

$$\Rightarrow L[\text{sen}\,wt] = \frac{\frac{w}{s^2}}{1 + \frac{w^2}{s^2}} = \frac{\frac{w}{s^2}}{\frac{s^2+w^2}{s^2}} \Rightarrow \boxed{L[\text{sen}\,wt] = \frac{w}{s^2+w^2}}$$

10) $L[f*g] = \int_0^{+\infty}\left(\int_0^{+\infty} f(t-\tau)g(\tau)\,d\tau\right)e^{-st}dt$ → mudança de variável: $p = t-\tau$
$dp = dt$

$$\int_0^{+\infty}\left(\int_0^{+\infty} f(p)g(\tau)\,d\tau\right)e^{-s(p+\tau)}dp = \int_0^{+\infty}\int_0^{+\infty} f(p)g(\tau)\,e^{-sp}\cdot e^{-s\tau}\,d\tau\,dp$$

$$\Rightarrow \int_0^{+\infty} f(p)e^{-sp}\int_0^{+\infty} g(\tau)e^{-s\tau}\,d\tau\,dp = \int_0^{+\infty} f(p)e^{-sp}dp \cdot \int_0^{+\infty} g(\tau)e^{-s\tau}\,d\tau$$

$$\Rightarrow \boxed{L[f*g] = L\left[\int_0^{+\infty} f(t-\tau)g(\tau)\,d\tau\right] = F(s)\cdot G(s)}$$

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11) $$Y(s) = L\left[\frac{7}{3}e^{-t} + \frac{3}{2}e^{-2t} - \frac{1}{6}e^{-4t}\right] \Longrightarrow Y(s) = \frac{7}{3}\frac{1}{s+1} + \frac{3}{2}\frac{1}{s+2} - \frac{1}{6}\frac{1}{s+4}$$

usando resultados dos exs. 01, 02 e 07

Impulso unitário $\delta(t) \rightarrow F(s) = 1 = U(s)$ (slide 16)

$$\Rightarrow F.T. \equiv G(s) = \frac{Y(s)}{U(s)} \Rightarrow \boxed{G(s) = \frac{7}{3}\frac{1}{s+1} + \frac{3}{2}\frac{1}{s+2} - \frac{1}{6}\frac{1}{s+4}}$$

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12) $$Y(s) = L\left[1 - \frac{7}{3}e^{-t} + \frac{3}{2}e^{-2t} - \frac{1}{6}e^{-4t}\right] \Longrightarrow Y(s) = \frac{1}{s} - \frac{7}{3}\frac{1}{s+1} + \frac{3}{2}\frac{1}{s+2} - \frac{1}{6}\frac{1}{s+4}$$

usando os resultados dos exs. 01, 02 e 07 e que $L[1] = \frac{1}{s}$ (slide 5, 6 e 16)

Degrau unitário $1(t) \rightarrow F(s) = \frac{1}{s} = U(s)$ (slide 16)

$$\Rightarrow F.T. \equiv G(s) = \frac{Y(s)}{U(s)} \Rightarrow \boxed{G(s) = 1 - \frac{7}{3}\frac{s}{s+1} + \frac{3}{2}\frac{s}{s+2} - \frac{1}{6}\frac{s}{s+4}}$$

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13) $$G_1(s) = \frac{K}{JL_a s^3 + (R_a J + BL_a)s^2 + (R_a B + KK_b)s}$$

polos: $JL_a s^3 + (R_a J + BL_a)s^2 + (R_a B + KK_b)s = 0$

$s\{JL_a s^2 + (R_a J + BL_a)s + (R_a B + KK_b)\} = 0$

$$s' = \frac{-(R_a J + BL_a) - \sqrt{(R_a J + BL_a)^2 - 4JL_a(R_a B + KK_b)}}{2JL_a}$$

$$s'' = \frac{-(R_a J + BL_a) + \sqrt{(R_a J + BL_a)^2 - 4JL_a(R_a B + KK_b)}}{2JL_a}$$

$s''' = 0$

$$G(s) = \frac{K}{Js^2 + Bs}$$

polos: $Js^2 + Bs = 0 \Rightarrow s(Js + B) = 0$

$$s' = -\frac{B}{J}$$

$s'' = 0$