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Exercícios da Aula 15/10

1) $L[Af(t)] = \int_0^\infty Af(t)e^{-st}dt = A\int_0^\infty f(t)e^{-st}dt \Rightarrow \boxed{L[Af(t)] = AF(s)}$

2) $L[f_1(t) \pm f_2(t)] = \int_0^\infty (f_1(t) \pm f_2(t))e^{-st}dt = \int_0^\infty f_1(t)e^{-st}dt \pm \int_0^\infty f_2(t)e^{-st}dt \Rightarrow \boxed{L[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)}$

3) $L[\int f(t)dt] = \int_0^\infty (\int f(t)dt \cdot e^{-st}dt) = -\frac{e^{-st}}{s}\int f(t)dt\Big|_0^\infty + \int_0^\infty \frac{e^{-st}}{s} f(t)\,dt$

Integração por partes: $\begin{cases} u = \int f(t)dt \\ dV = e^{-st}dt \end{cases} \rightarrow \begin{cases} du = f(t)dt \\ V = -\frac{e^{-st}}{s} \end{cases}$

$\Rightarrow \boxed{L[\int f(t)dt] = \frac{F(s)}{s} + \frac{1}{s}[\int f(t)dt]_{t=0}}$

4) $L[e^{-at}f(t)] = \int_0^\infty e^{-at}f(t)e^{-st}dt = \int_0^\infty e^{-(a+s)t}f(t)\,dt \Rightarrow \boxed{L[e^{-at}f(t)] = F(s+a)}$

5) $L[f(\frac{t}{a})] = \int_0^\infty f(\frac{t}{a})e^{-st}dt \underset{\substack{u=\frac{t}{a} \\ du=\frac{dt}{a}}}{=} \int_0^\infty f(u)e^{-sua}\,a\,du = a\int_0^\infty f(u)e^{-(as)u}du \Rightarrow \boxed{L[f(\frac{t}{a})] = aF(as)}$

6) Teorema do Valor Final: $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

$L[\frac{df}{dt}] = sF(s) - f(0) \Rightarrow \lim_{s\to\infty} L[\frac{df}{dt}] = \lim_{s\to\infty}(sF(s) - f(0))$

$\lim_{s\to\infty}\int_0^\infty \frac{df}{dt}e^{-st}dt = \int_0^\infty \frac{df}{dt}(\lim_{s\to\infty} e^{-st}) \xrightarrow{} 0\, dt = 0$

$\lim_{s\to\infty}[sF(s) - f(0)] = \lim_{s\to\infty} sF(s) - \lim_{s\to\infty} f(0) = 0$

$\lim_{s\to\infty} f(0) = \lim_{t\to 0} f(t) \Rightarrow \boxed{\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)}$ Teorema do Valor Inicial

7) $L[e^{-at}] = \int_0^\infty e^{-at}e^{-st}dt = \int_0^\infty e^{-(a+s)t}dt = \frac{e^{-(a+s)t}}{-(a+s)}\Big|_0^\infty = \lim_{t\to\infty}\left(\cancel{\frac{e^{-(a+s)t}}{-(a+s)}}\right) - \frac{e^{-(a+s)\cdot 0}}{-(a+s)} \Rightarrow$

$\Rightarrow \boxed{L[e^{-at}] = \frac{1}{s+a}}$

8) $L[te^{-at}] = \int_0^\infty te^{-at}\cdot e^{-st}dt = \int_0^\infty te^{-(a+s)t}dt$

Integração por partes: $\begin{cases} u = t \rightarrow du = dt \\ dV = e^{-(a+s)t}dt \rightarrow V = \frac{e^{-(a+s)t}}{-(a+s)} \end{cases}$

$L[te^{-at}] = \cancel{\frac{t\cdot e^{-(a+s)t}}{-(a+s)}}\Big|_0^\infty - \int_0^\infty \frac{e^{-(a+s)t}}{-(a+s)}dt \Rightarrow$

$\Rightarrow \boxed{L[te^{-at}] = \frac{1}{(s+a)^2}}$

9) $L[\sin\omega t] = \int_0^\infty \sin\omega t\, e^{-st}\,dt \rightarrow$ por partes $\begin{cases} u = \sin\omega t \\ dV = e^{-st}dt \end{cases} \Rightarrow \begin{cases} du = \omega\cos\omega t \\ V = -\frac{e^{-st}}{s} \end{cases}$

$L[\sin\omega t] = \cancel{\sin\omega t\left(-\frac{e^{-st}}{s}\right)\Big|_0^\infty} - \int_0^\infty \omega\cos\omega t\left(-\frac{e^{-st}}{s}\right)dt = \int_0^\infty \omega\cos\omega t\,\frac{e^{-st}}{s}\,dt =$

$= \omega\cos\omega t\left(-\frac{e^{-st}}{s^2}\right)\Big|_0^\infty - \int_0^\infty \omega^2\sin\omega t\,\frac{e^{-st}}{s^2}\,dt = \frac{\omega}{s^2} - \frac{\omega^2}{s^2}\int_0^\infty \sin\omega t\, e^{-st}\,dt$

$\Rightarrow \boxed{L[\sin\omega t] = \frac{\omega}{s^2+\omega^2}}$

10) $L[f*g] = \int_0^\infty (f*g)e^{-st}\,dt = \int_0^\infty e^{-st}\left(\int_0^\infty f(t-\tau)g(\tau)\,d\tau\right)dt = \int_0^\infty\int_0^\infty f(t-\tau)g(\tau)e^{-st}\,dt\,d\tau$

$\alpha = t-\tau \rightarrow d\alpha = dt$; $\int_0^\infty\int_0^\infty f(\alpha)g(\tau)e^{-s(\alpha+\tau)}\,d\alpha\,d\tau = \int_0^\infty f(\alpha)e^{-s\alpha}\,d\alpha \int_0^\infty g(\tau)e^{-s\tau}\,d\tau$

$\Rightarrow \boxed{L[f*g] = F(s)\,G(s)}$

11) $Y(t) = \frac{7}{3}e^{-t} + \frac{3}{2}e^{-2t} - \frac{1}{6}e^{-4t}$, $L[e^{-at}] = \frac{1}{a+s}$

$L[Y(t)] = Y(s) = \frac{7}{3}\cdot\frac{1}{s+1} + \frac{3}{2}\cdot\frac{1}{s+2} - \frac{1}{6}\cdot\frac{1}{s+4}$

Impulso: $u(t) = \delta(t) \xrightarrow{L} U(s) = 1$

$G(s) = \frac{Y(s)}{U(s)} = \frac{Y(s)}{1} \Rightarrow \boxed{G(s) = \frac{7}{3}\frac{1}{s+1} + \frac{3}{2}\frac{1}{s+2} - \frac{1}{6}\frac{1}{s+4}}$

12) $Y(t) = 1 - \frac{7}{3}e^{-t} + \frac{3}{2}e^{-2t} - \frac{1}{6}e^{-4t}$

$Y(s) = \frac{1}{s} - \frac{7}{3}\frac{1}{s+1} + \frac{3}{2}\frac{1}{s+2} - \frac{1}{6}\frac{1}{s+4}$

Degrau: $u(t) = H(t) \xrightarrow{L} U(s) = \frac{1}{s}$

$G(s) = \frac{Y(s)}{U(s)} = \frac{Y(s)}{1/s} \Rightarrow \boxed{G(s) = 1 - \frac{7}{3}\frac{s}{s+1} + \frac{3}{2}\frac{s}{s+2} - \frac{1}{6}\frac{s}{s+4}}$

13) Motor c.c.: $\begin{cases} L_a D i_a + R_a i_a = e_a - K_b\omega \\ J\ddot{\theta} + B\dot{\theta} = K i_a \end{cases} \xrightarrow{L} \begin{cases} (L_a s + R_a) I_a = E_a - K_b s\Theta \\ (Js^2 + Bs)\Theta = K I_a \end{cases}$

$Y(s) = \Theta(s)$ e $U(s) = E_a(s) \rightarrow G_1(s) = \frac{\Theta(s)}{E_a(s)} \Rightarrow \boxed{G_1(s) = \frac{K}{JL_as^3 + (R_aJ + BL_a)s^2 + (R_aB + KK_b)s}}$

Polos do sistema: $E_a(s) = 0$

$s\left(JL_as^2 + (R_aJ + BL_a)s + R_aB + KK_b\right) = 0 \Rightarrow s = 0$

$JL_as^2 + (R_aJ + BL_a)s + R_aB + KK_b = 0 \Rightarrow s = \frac{-(R_aJ + BL_a) \pm \sqrt{(R_aJ+BL_a)^2 - 4JL_a(R_aB + KK_b)}}{2JL_a}$

$$\boxed{\begin{aligned} s_1 &= 0 \\ s_2 &= \frac{-(R_aJ + BL_a) + \sqrt{(R_aJ+BL_a)^2 - 4JL_a(R_aB + KK_b)}}{2JL_a} \\ s_3 &= \frac{-(R_aJ + BL_a) - \sqrt{(R_aJ+BL_a)^2 - 4JL_a(R_aB + KK_b)}}{2JL_a} \end{aligned}}$$