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1. $\mathcal{L}(AF(t)) = \int_0^{\infty} Af(t) e^{-st} dt = A\int_0^{\infty} f(t) e^{-st} dt = AF(s)$

2- $\mathcal{L}(f_1(t) \pm f_2(t)) = \int_0^{\infty} (f_1(t) \pm f_2(t)) e^{-st} dt = \int_0^{\infty} f_1(t) e^{-st} \pm \int_0^{\infty} f_2(t) e^{-st} dt$

$= F_1(s) \pm F_2(s)$

3. $\mathcal{L}(\int f(t) dt) = \int_0^{\infty} (\int f(t) dt\, e^{-st} dt)$

$u = \int f(t) dt \quad du = f(t) dt \rightarrow \quad = \left[\int f(t) dt \cdot \left(\frac{e^{-st}}{s}\right)\right]_0^{\infty} - \int -\frac{e^{-st}}{s} \cdot f(t)$

$dv = e^{-st} dt \quad v = -\frac{e^{-st}}{s} \quad = \frac{1}{s}\left((\int f(t) dt)_{t=0} + F(s)\right)$

4- $\mathcal{L}(e^{-at} f(t)) = \int_0^{\infty} f(t) e^{-(s+a)t} dt = F(s+a)$

5- $\mathcal{L}(f(\frac{t}{a})) = \int_0^{\infty} f(\frac{t}{a}) \cdot e^{-st} dt$

$u = \frac{t}{a}$

$du = \frac{dt}{a} \rightarrow dt = a\,du \rightarrow \int_0^{\infty} f(u) e^{-aus} a\,du = a\int_0^{\infty} f(a) e^{-asu} = aF(sa)$

6- $\lim f(t) = \lim_{s \to \infty} sF(s) \qquad \mathcal{L}(\dot{f}(t)) = sF(s) - f(0)$

$\lim_{s \to \infty} \int_0^{\infty} \dot{f}(t) e^{-st} dt = \lim_{s \to \infty} (sF(s) - f(0^+)) = \int_0^{\infty} \dot{f}(t) \lim_{s \to \infty} e^{-st} dt = \lim_{s \to \infty} (sF(s) - f(0^+))$

$\lim_{s \to \infty} sF(s) = \lim_{s \to \infty} f(0^+) \quad Cte$

$\lim_{s \to \infty} sF(s) = f(0^+)$

7- $\mathcal{L}(e^{-at}) = \int_0^\infty e^{-at} e^{-st} dt = \int_0^\infty e^{-(s+a)t} dt$

$$= -\frac{e^{-(s+a)t}}{s+a}\Bigg]_0^\infty = \frac{1}{s+a}$$

8- $\mathcal{L}(te^{-at}) = \int_0^\infty te^{-at} e^{-st} = \int_0^\infty te^{-(a+s)t} dt = t$

$u = t \quad du = dt$

$dv = e^{-(a+s)t} \quad v = e^{-(a+s)t}$

$$\rightarrow \frac{te^{-(a+s)t}}{-a+s}\Bigg|_0^\infty - \int_0^\infty \frac{e^{-(a+s)t}}{-(a+s)}$$

$$= \frac{1}{(s+a)^2}$$

9- $y = \mathcal{L}(\sin\omega t) = \int_0^\infty \sin\omega t\, e^{-st} dt$

$y = -\frac{1}{s}e^{-st}\sin\omega t - \int_0^\infty \omega\cos\omega t\left(\frac{-1}{s}e^{-st}\right)dt = \frac{e^{-st}}{s}\sin\omega t + \frac{\omega}{s}\int_0^\infty \cos\omega t \frac{e^{-st}}{s} dt$

$y = \int_0^\infty \sin\omega t e^{-st} dt$

$\int_0^\infty \sin\omega t e^{-st} dt \left(1 + \frac{\omega^2}{s^2}\right) = \frac{\omega}{s^2+\omega^2}$

10- $\mathcal{L}(f*g) = \int_0^\infty (f*g) e^{-st} dt = \int_0^\infty e^{-st} \left(\int_0^t f(t-\tau) g(\tau) d\tau\right) dt$

$= \int_0^\infty \int_0^\infty f(t-\tau) g(\tau) e^{-st} dt\, d\tau$

$u = t - \tau$

$du = dt$

$\mathcal{L}(f*g) = \int_0^\infty \int_0^\infty f(u) g(\tau) e^{-s(u+\tau)} du\, d\tau$

$= \underbrace{\left(\int_0^\infty f(u) e^{-su} du\right)}_{F(s)} \cdot \underbrace{\left(\int_0^\infty g(\tau) e^{-s\tau} d\tau\right)}_{G(s)}$

$\mathcal{L}(f*g) = F(s)G(s)$

11. $y(t) = \frac{7}{3}e^{-t} + \frac{3}{2}e^{-2t} - \frac{1}{6}e^{-4t}$

$Y(s) = \frac{7}{3}\frac{1}{s+1} + \frac{3}{2}\cdot\frac{1}{s+2} - \frac{1}{6}\frac{1}{s+4}$ $\quad u(t) = \delta(t)$

$U(s) = 1$

$G(s) = \frac{Y(s)}{U(s)} \quad \frac{Y(s)}{1} \rightarrow G(s) = Y(s)$

$G(s) = \frac{7}{3(s+1)} + \frac{3}{2(s+2)} - \frac{1}{6(s+4)}$

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12. $y(t) = 1 - \frac{7}{3}e^{-t} + \frac{3}{2}e^{-2t} - \frac{1}{6}e^{-4t}$

$Y(s) = \frac{1}{s} - \frac{7}{3}\frac{1}{s+1} + \frac{3}{2}\frac{1}{s+2} - \frac{1}{6}\frac{1}{s+4}$

$u(t) = 1 \quad U(s) = \frac{1}{s}$

$G(s) = \frac{Y(s)}{U(s)} \qquad G(s) = 1 - \frac{7s}{3(s+1)} + \frac{3s}{2(s+2)} - \frac{s}{6(s+4)}$

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13. $G_1(s) = \frac{K}{JL_0 s^3 + (R_0 J + BL_0)s^2 + (R_0 B + KK_b)s}$

$JL_0 s^3 + (R_0 J + BL_0)s^2 + (R_0 B + KK_b)s = 0$

$s_1 = 0$

$s_2 = -\frac{(BL_0) + JR_0 + \sqrt{(B^2L_0^2 - 2JL_0(BR_0) + 2KK_b) + J^2R^2}}{2JL_0}$

$s_3 = \frac{\sqrt{B^2L_0^2 - 2JL_0(BR_0 + 2KK_b) + J^2R^2} - JR_0 - BL_0}{2JL_0}$