PME-3380

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① $L[A f(t)] = \int_{-\infty}^{\infty} A f(t) e^{-st} dt = A \int_{-\infty}^{\infty} f(t) e^{-st} dt \Rightarrow \boxed{L[A f(t)] = A F(s)}$

② $L[f_1(t) \pm f_2(t)] = \int (f_1(t) \pm f_2(t)) e^{-st} dt \Rightarrow \boxed{L[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)}$

③ $L[\int_0^t F(t) dt] = \int_t^{\infty} e^{-st} dt \int_0^t f(t) dt = -\frac{e^{-st}}{s} \Big|_t^{\infty} \int_0^t f(t) dt = \frac{1}{s} \int_0^t f(t) dt\, e^{-st} \Rightarrow$

$\Rightarrow \boxed{L[\int_0^t f(t) dt] = f(s)/s}$

④ $L[e^{-\alpha t} f(t)] = \int e^{-\alpha t} e^{-st} f(t) dt = \int e^{-(\alpha+s)t} f(t) dt \Rightarrow \boxed{L[e^{-\alpha t} f(t)] = F(s+\alpha)}$

⑤ $L[f(\frac{t}{a})] = L[f(U)] = \int f(U) e^{-sUa} dUa \Rightarrow \boxed{L[f(\frac{t}{a})] = aF(sa)}$

⑥ $\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s) \Rightarrow L[\dot{f}(t)] = sF(s) - f(0) \Rightarrow \lim_{s \to \infty} [\dot{f}(t)] = \lim_{s \to \infty} sF(s) - F(0)$

$\int_0^{\infty} \dot{f}(t) \lim_{s \to \infty} e^{-st} dt = 0 = \lim_{s \to \infty} sF(s) - F(0) \Rightarrow \boxed{\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)}$

⑦ $L[e^{-at}] = \int_0^{\infty} e^{-at} e^{-st} dt = \int_0^{\infty} e^{-(a+s)t} dt = \frac{-1}{a+s} e^{-(a+s)t} \Big|_0^{\infty} \Rightarrow \boxed{L[e^{-at}] = \frac{1}{a+s}}$

⑧ $L[t \cdot e^{-at}] = \int_0^{\infty} t e^{-(s+a)t} dt = \int_0^{\infty} g(t) e^{-st} dt = G(s) = -\frac{\partial F(s)}{\partial s} \Rightarrow \boxed{L[t \cdot e^{-at}] = \frac{1}{(a+s)^2}}$

⑨ $y = L[\operatorname{sen} \omega t] = \int_0^{\infty} \operatorname{sen} \omega t\, e^{-st} dt \Rightarrow \begin{matrix} U' = e^{-st} \\ U = -\frac{1}{s} e^{-st} \end{matrix} \Rightarrow \begin{matrix} V' = \omega \cos \omega t \\ V = \operatorname{sen} \omega t \end{matrix}$

$y = -\frac{1}{s} e^{-st} \operatorname{sen} \omega t - \int_0^{\infty} \omega \cos \omega t \left(-\frac{1}{s} e^{-st}\right) dt = -\frac{e^{-st}}{s} \operatorname{sen} \omega t + \frac{\omega}{s} \int_0^{\infty} \cos \omega t \frac{e^{-st}}{s} dt$

$y = -\frac{e^{-st}}{s} \operatorname{sen} \omega t + \frac{\omega}{s} \left[-\frac{e^{-st}}{s} \cos \omega t - \int_0^{\infty} \frac{e^{-st}}{s} (\omega \operatorname{sen} \omega t) dt \right]$

$y = -\frac{e^{-st}}{s} \operatorname{sen} \omega t - \frac{\omega}{s^2} e^{-st} \cos \omega t - \frac{\omega^2}{s^2} \int_0^{\infty} (e^{-st} \operatorname{sen} \omega t) dt = \int_0^{\infty} \operatorname{sen} \omega t\, e^{-st} dt$

$\left(\int_0^{\infty} \operatorname{sen} \omega t\, e^{-st} dt \right) \left(1 + \frac{\omega^2}{s^2} \right) = \left[-e^{-st} \left(\frac{1}{s} \operatorname{sen} \omega t + \frac{\omega}{s^2} \cos \omega t \right) \right]_0^{\infty} = 0 - (-1)\left(0 + \frac{\omega}{s^2}\right) = \frac{\omega}{s^2}$

$\int_0^{\infty} \operatorname{sen} \omega t\, e^{-st} dt = \frac{s^2}{s^2 + \omega^2} \cdot \frac{\omega}{s^2} = \boxed{\frac{\omega}{s^2 + \omega^2}}$

⑩ $L[\int_0^{\infty} f(t-\tau) g(\tau) d\tau] = L[\int_0^t f(\eta) g(t-\eta) d\eta] = y = \int_0^{\infty} f(\eta) \int_{\eta}^{\infty} g(t-\eta) e^{-st} \partial t \partial \eta =$

$= \int_0^{\infty} f(\eta) \int_0^{\infty} g(U) e^{-s(U+\eta)} \partial U \partial \eta = \int_0^{\infty} f(\eta) e^{-s\eta} \partial \eta \int_0^{\infty} g(U) e^{-sU} \partial U \Rightarrow \boxed{y = F(s) G(s)}$

⑪ $Y(s) = G(s) \cdot U(s) = G(s) \cdot 1 \Rightarrow \boxed{Y(s) = \frac{7}{3}\left(\frac{1}{s+1}\right) + \frac{3}{2}\left(\frac{1}{s+2}\right) + \frac{1}{6}\left(\frac{1}{s+4}\right) = G(s)}$

⑫ $\frac{\partial y_2(t)}{\partial t} = \frac{7}{3} e^{-t} - 3e^{-2t} + \frac{4}{6} e^{-4t} = y_1(t) \Rightarrow \boxed{G(s) = \frac{7}{3}\left(\frac{1}{s+1}\right) - 3\left(\frac{1}{s+2}\right) + \frac{4}{6}\left(\frac{1}{s+4}\right)}$

⑬ $G_1(s) = \frac{K}{J L_a s^3 + (R_a J + B L_a) s^2 + (R_a B + K K_1) s}$

$J L_a s^3 + (R_a J + B L_a) s^2 + (R_a B + K K_b) s = 0 \Rightarrow \boxed{s_1 = 0}$

$$\boxed{s_2 = \frac{-\sqrt{B^2 L_a^2 - 2 J L_a (B R_a + 2 K K_b) + J^2 R^2} + B L_a + J R_a}{2 J L_a}}$$

$$\boxed{s_3 = \frac{\sqrt{B^2 L_a^2 - 2 J L_a (B R_a + 2 K K_b) + J^2 R^2} - B L_a - J R_a}{2 J L_a}}$$