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1) $\mathcal{L}[Af(t)] = \int_0^\infty Af(t) \cdot e^{-st} dt = A\int_0^\infty f(t) \cdot e^{-st} dt = \underline{AF(s)}$ //

2) $\mathcal{L}[f_1(t) \pm f_2(t)] = \int_0^\infty (f_1(t) \pm f_2(t)) \cdot e^{-st} dt = \int_0^\infty f_1(t) \cdot e^{-st} \pm \int_0^\infty f_2(t)\, e^{-st}$

$= \underline{F_1(s) \pm F_2(s)}$ //

3) $\mathcal{L}[\int f(t) dt] = \int_0^\infty \int f(t) dt \cdot e^{-st} dt$

$u = \int f(t) dt \qquad dv = e^{-st} dt$

$du = f(t) \qquad v = -\frac{e^{-st}}{s}$

$uv - \int v du \Rightarrow \left[\int f(t) dt \cdot \left(-\frac{e^{-st}}{s}\right)\right]_0^\infty - \int_0^\infty -\frac{e^{-st}}{s} \cdot f(t) =$

$= \left[(\int f(t) dt)_\infty \cdot \cancel{-\frac{e^{-\infty t}}{s}}^{\,0} + (\int f(t) dt)_0 \cdot \frac{1}{s}\right] + \frac{F(s)}{s} =$

$= \underline{\frac{1}{s}[\int f(t) dt]_{t=0} + \frac{1}{s} \cdot F(s)}$ //

④ $\mathcal{L}[e^{-at}f(t)] = \int_0^\infty f(t)\cdot e^{-(s+a)t}\, dt = \underline{F(s+a)}$ //

⑤ $\mathcal{L}\left[f\left(\frac{t}{a}\right)\right] = \int_0^\infty f\left(\frac{t}{a}\right)\cdot e^{-st}\, dt$

. Substituindo: $u = \frac{t}{a}$ $du = \frac{dt}{a} \rightarrow dt = a\,du$

$\int_0^\infty f(u)\cdot e^{-aus}\, a\,du = a\int_0^\infty f(u)\cdot e^{-as\cdot u} = \underline{a\ F(sa)}$ //

⑥ $\lim\limits_{t\to 0^+} f(t) = \lim\limits_{s\to\infty} s\,F(s)$

$\mathcal{L}\{\dot{f}(t)\} = sF(s) - f(0^+)$

$\lim\limits_{s\to\infty}\int_0^\infty \dot{f}(t)\,e^{-st}\,dt = \lim\limits_{s\to\infty}(sF(s) - f(0^+))$

$\int_0^\infty \dot{f}(t)\cdot\lim\limits_{s\to\infty} \underbrace{e^{-st}}_{0}\,dt = \lim\limits_{s\to\infty}(sF(s) - f(0^+))$

$\lim\limits_{s\to\infty} sF(s) = \lim\limits_{s\to\infty} f(0^+)$ $\rightarrow$ constante

$\underline{\lim\limits_{s\to\infty} sF(s) = f(0^+)}$ //

⑦ $\mathcal{L}(e^{-at}) = \int_0^{\infty} e^{-at} \cdot e^{-st}\, dt = \int_0^{\infty} e^{-(a+s)t}\, dt$

$$= -\frac{e^{-(a+s)t}}{a+s}\Big]_0^{\infty} = \frac{1}{a+s}$$

⑧ $\mathcal{L}(te^{-at}) = \int_0^{\infty} te^{-at} \cdot e^{-st}\, dt = \int_0^{\infty} t \cdot e^{-(a+s)t}\, dt$

$u = t \qquad dv = e^{-a+s}$

$du = 1 \qquad v = -e^{-a+s}/(a+s)$

$$-\frac{te^{-a+s}}{a+s}\Big|_0^{\infty} - \int_0^{\infty} \frac{-e^{-a+s}}{a+s}\, dt = -\left(\frac{+e^{-(a+s)}}{(a+s)^2}\right)\Big|_0^{\infty} = \frac{1}{(a+s)^2}$$

⑨ $sen(wt) = \int_0^{\infty} sen(wt) \cdot e^{-st}\, dt$

$u = sen(wt) \qquad dv = e^{-st}$

$du = w\cos(wt) \qquad v = -\frac{e^{-st}}{s}$

$$-\frac{e^{-st}}{s} \cdot \overset{\nearrow 0}{sen(wt)}\Big|_0^{\infty} - \int_0^{\infty} -\frac{e^{-st}}{s} \cdot w\cos(wt)\, dt =$$

$u = \cos(wt) \qquad dv = e^{st}$

$du = wsen(wt) \qquad v = e^{st}/s$

$$-\frac{e^{-st}}{s^2} w\cos wt\Big|_0^{\infty} - \int_0^{\infty} \frac{e^{-st}}{s^2} \cdot w^2 \cdot sen\, wt\, dt$$

$$\frac{w}{s^2} - \frac{w^2}{s^2} \cdot \int_0^{\infty} sen\, wt \cdot e^{-st}\, dt$$

Femmina

$$F(s) = \frac{\omega}{s^2} - \frac{\omega^2}{s^2} F(s)$$

$$F(s) = \frac{\omega}{s^2} \Big/ \frac{1+\omega^2}{s^2} = \frac{\omega}{s^2+\omega^2}$$

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$$\mathcal{L}(f*g) = \int_0^\infty (f*g)\, e^{-st}\, dt =$$

$$= \int_0^\infty e^{-st} \left(\int_0^\infty f(t-z)\, g(z)\, dz \right) dt =$$

$$= \int_0^\infty \int_0^\infty f(t-z)\, g(z)\, e^{-st}\, dt\, dz$$

$$= \int_0^\infty \int_0^\infty f(\gamma)\, g(z)\, e^{-s(\gamma+z)}\, d\gamma\, dz$$

$$= \int_0^\infty f(\gamma)\, e^{-s\gamma}\, d\gamma \cdot \int_0^\infty g(z) \cdot e^{-sz}\, dz$$

$$= F(s)\, G(s)$$

①

$$y(t) = \frac{7}{3}e^{-t} + \frac{3}{2}e^{-2t} - \frac{1}{6}e^{-4t}$$

$$Y(s) = \frac{7}{3} \cdot \frac{1}{s+1} + \frac{3}{2}\frac{1}{s+2} - \frac{1}{6}\frac{1}{s+4}$$

$$G(s) = \frac{Y(s)}{U(s)}_{=1} = \frac{7}{3}\frac{1}{s+1} + \frac{3}{2}\frac{1}{s+2} - \frac{1}{6}\frac{1}{s+4}$$

②

$$y(t) = 1 - \frac{7}{3}e^{-t} + \frac{3}{2}e^{-2t} - \frac{1}{6}e^{-4t}$$

$$G = \frac{Y(s)}{U(s)_{=\frac{1}{s}}} = s\left(\frac{1}{s} - \frac{7}{3}\frac{1}{s+1} + \frac{3}{2}\frac{1}{s+2} - \frac{1}{6}\frac{1}{s+4}\right)$$

③

$$G(s) = \frac{K}{JL_as^3 + (R_aJ + BL_a)s^2 + (R_aB + KK_b)s}$$

$s = 0$

$$JL_as^2 + (R_aJ + BL_a)s + (R_aB + KK_b) = 0$$

$$s = \frac{-(R_aJ + BL_a) \pm \sqrt{(R_aJ \pm BL_a)^2 - 4JL_a \cdot (R_aB + KK_b)}}{2 \cdot JL_a}$$