Nome : Wallace Moreira e Silva
Número USP : 10823772
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Exercício da aula 15/10/2020

1) $\mathcal{L}(AF(t)) = \int_0^\infty A f(t) e^{-st} dt = A \int_0^\infty f(t) e^{-st} dt = AF(s)$

2) $\mathcal{L}\{F_1(t) \pm F_2(t)\} = \int_0^\infty (f_1(t) \pm f_2(t)) e^{-st} dt =$

$\int_0^\infty f_1(t) e^{-st} dt \pm \int_0^\infty f_2(t) e^{-st} dt = F_1(s) \pm F_2(s)$

3) $\mathcal{L}\left\{ \int f(t) dt \right\} = \int_0^\infty \left(\int f(t) dt \, e^{-st} dt \right)$

$\begin{cases} u = \int f(t) dt \\ dv = e^{-st} dt \end{cases} \quad \begin{cases} du = f(t) dt \\ v = -\dfrac{e^{-st}}{s} \end{cases}$

$= \frac{1}{2} \int f(t) dt + F(s)$

4) $\mathcal{L}\{e^{-at} F(t)\} = \int_0^\infty e^{-at} . f(t) . e^{-st} dt$

$= \int_0^\infty f(t) e^{-(a+s)} dt = F(a+s)$

5) $\mathcal{L}\left\{F(t/a)\right\} = \int_0^\infty f(t/a)\, e^{-st}\, dt$

$\begin{cases} u = t/a \\ du = dt/a \end{cases} \quad \therefore \int_0^\infty f(u)\, e^{-sau}\, a\, du$

$= a \int_0^\infty f(u) \cdot e^{-(sa)u}\, du = a F(as)$

6) $\lim_{t \to \infty} F(t) = \lim_{s \to 0} sF(s)$

$\lim_{s \to 0} \mathcal{L}\left(dF/dt\right) = \lim_{s \to 0} \left(sF(s) - F(0)\right)$

$\lim_{s \to 0} \left[\int_0^\infty dF/dt\, e^{-st}\, dt\right] = -F(0) + \lim_{s \to 0} sF(s)$

$\int_0^\infty \frac{dF}{dt} \cdot \lim_{s \to 0} e^{-st}\, dt = F\infty - F(0) = -F(0) + \lim_{s \to 0} sF(s)$

$\lim_{t \to \infty} F(t) = \lim_{s \to 0} sF(s)$

7) $F(t) = e^{-at} \rightarrow \mathcal{L}\{e^{-at}\}$

$$= \int_0^{\infty} e^{-st} \cdot e^{-at}\, dt = \int_0^{\infty} e^{-(a+s)t}\, dt = \left. \frac{-e^{-(a+s)t}}{(a+s)} \right|_0^{\infty}$$

$$= \lim_{t \to \infty} \frac{-e^{(a+s)t}}{(a+s)} + \frac{e^{-(a+s) \cdot 0}}{(a+s)} = \frac{1}{a+s}$$

8) $\mathcal{L}[t \cdot e^{at}] = \int_0^{\infty} t\, e^{-(s+a)t}\, dt = \int_0^{\infty} g(t)\, e^{-st}\, dt$

$$G(s) = -\frac{\partial F(s)}{\partial s} = \frac{1}{(a+s)^2}$$

9) $y = \mathcal{L}[\sin \omega t] = \int_0^{\infty} \sin \omega t\, e^{-st}\, dt$

$$y = -\frac{1}{s} e^{-st} \sin \omega t - \int_0^{\infty} \omega \cos \omega t \left(-\frac{1}{s} e^{-st}\right) dt$$

$$y = \frac{e^{-st}}{s} \sin \omega t + \frac{\omega}{s} \int_0^{\infty} \cos \omega t\, \frac{e^{-st}}{s}\, dt$$

$$y = \int_0^{\infty} \sin \omega t\, e^{-st}\, dt \Rightarrow \int_0^{\infty} \sin \omega t\, e^{-st}\, dt \cdot \left(1 + \frac{\omega^2}{s^2}\right)$$

$$= \frac{\omega}{s^2 + \omega^2}$$

10) $y = \mathcal{L}\left[\int_0^\infty f(t-\tau)\, g(\tau)\, d\tau\right] = \mathcal{L}\left[\int_0^t f(\tau)\, g(t-\tau)\, d\tau\right]$

$$y = \int_0^\infty f(\tau) \int_\tau^\infty g(t-\tau)\, e^{-st}\, dt\, d\tau = \int_0^\infty f(\tau) \int_0^\infty g(u)\, e^{-s(u+\tau)}\, du\, d\tau$$

$$y = \int_0^\infty f(\tau)\, e^{-s\tau}\, d\tau \int_0^\infty g(u)\, e^{-su}\, du = F(s)\, G(s)$$

11) $Y(s) = G(s) \cdot U(s) = G(s) \cdot 1$

$$Y(s) = \frac{7}{3}\left(\frac{1}{s+1}\right) + \frac{3}{2}\left(\frac{1}{s+2}\right) + \frac{1}{6}\left(\frac{1}{s+4}\right) = G(s)$$

12) $\dfrac{dy_2(t)}{dt} = \dfrac{7}{3} e^{-t} - 3e^{-2t} + \dfrac{4}{6} e^{-4t} = y_1(t)$

$$G(s) = \frac{7}{3}\left(\frac{1}{s+1}\right) - 3\left(\frac{1}{s+2}\right) + \frac{4}{6}\left(\frac{1}{s+4}\right)$$

13) $G_1(s) = \dfrac{K}{J L_a s^3 + (R_a J + B L_a)s^2 + (R_a B + K K_1)s}$

$$J L_a s^3 + (R_a J + B L_a)s^2 + (R_a B + K K_b)s = 0$$

$$\begin{cases} s_1 = 0 \\ s_2 = -\dfrac{B L_a + J R_a + \sqrt{B^2 L_a^2 - 2 J L_a (B R_a) + 2 K K_b) + J^2 R^2}}{2 J L_a} \\ s_3 = \dfrac{\sqrt{B^2 L_a^2 - 2 J L_a (B R_a + 2 K K_b) + J^2 R^2} - J R_a - B L_a}{2 J L_a} \end{cases}$$