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PME 3380 - EXERCÍCIOS AULA 15/10

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$$\mathcal{L}[f(t)] = \int_0^\infty f(t)e^{-st}dt = F(s)$$

1. $\mathcal{L}[Af(t)] = \int_0^\infty Af(t)e^{-st}dt = A\int_0^\infty f(t)e^{-st}dt = AF(s)$

2. $\mathcal{L}[f_1(t) \pm f_2(t)] = \int_0^\infty (f_1(t) \pm f_2(t))e^{-st}dt = \int_0^\infty f_1(t)e^{-st}dt \pm \int_0^\infty f_2(t)e^{-st}dt$

$= F_1(s) \pm F_2(s)$

3. $\mathcal{L}[\int f(t)dt] = \int_0^\infty \left(\int f(t)dt\right)e^{-st}dt \overset{*}{=} -\frac{e^{-st}}{s}\int f(t)dt\Big|_0^\infty + \int \frac{e^{-st}}{s}f(t)dt$

$\left(\begin{array}{lll} * \; u = \int f(t)dt & \rightarrow & du = f(t)dt \\ v = e^{-st}dt & \rightarrow & v = -e^{-st}/s \end{array} \right)$

$= \frac{1}{s}\left[\int f(t)dt\right]_{t=0} + \frac{F(s)}{s}$

4. $\mathcal{L}[e^{-at}f(t)] = \int_0^\infty e^{-at}\cdot f(t)\cdot e^{-st}dt = \int_0^\infty f(t)e^{-(a+s)t}dt = F(a+s)$

5. $\mathcal{L}[f(t/a)] = \int_0^\infty f(t/a)e^{-st}dt \overset{*}{=} \int_0^\infty f(u)e^{-sau}a\,du = aF(as)$

$*\; u = t/a \;;\; du = 1/a\, dt$

6. teorema do valor inicial: $\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s)$

como $\mathcal{L}_+[df(t)/dt] = sF(s) - f(0^+)$:

$\lim_{s\to\infty}\int_{0^+}^\infty \frac{df(t)}{dt}e^{-st}dt = \lim_{s\to\infty}[sF(s) - f(0^+)]$

$\int_{0^+}^\infty \frac{df(t)}{dt}\left(\cancel{\lim_{s\to\infty} e^{-st}}\right)^{0}dt = \lim_{s\to\infty}[sF(s) - f(0^+)] = \lim_{s\to\infty} sF(s) - \lim_{s\to\infty} f(0^+)$

como $\lim_{s\to\infty} f(0^+) = f(0^+) = \lim_{t\to 0^+} f(t)$:

$\lim_{t\to\infty} f(t) = \lim_{s\to\infty} sF(s)$

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7. $\mathcal{L}[e^{-at}] = \int_0^\infty e^{-at} e^{-st} dt = \int_0^\infty e^{-(a+s)t} dt = -\frac{e^{-(a+s)t}}{a+s}\Bigg|_0^\infty$

$= \lim_{t\to\infty}\left(\frac{-e^{-(a+s)t}}{a+s}\right) + \frac{e^{-(a+s)\cdot 0}}{a+s} = \frac{1}{a+s}$ //

8. $\mathcal{L}[te^{-at}] = \int_0^\infty te^{-at} e^{-st} = \int_0^\infty te^{-(a+s)t} dt \overset{*}{=} t$

$\left(\begin{array}{l} * \; u = t \to du = dt \\ dv = e^{-(a+s)t} \to v = \frac{e^{-(a+s)t}}{-(a+s)} \end{array}\right)$

$= t\frac{e^{-(a+s)t}}{-(a+s)}\Bigg|_0^\infty - \int_0^\infty \frac{e^{-(a+s)t}}{-(a+s)} dt = \frac{1}{(s+a)^2}$ //

9. $\mathcal{L}[sen(\omega t)] = \int_0^\infty sen(\omega t) e^{-st} dt \overset{*}{=} \cancel{-\frac{sen(\omega t) e^{-st}}{s}}^{\,0}\Bigg|_0^\infty + \int_0^\infty \frac{\omega}{s} e^{-st} cos(\omega t) dt$

$\left(\begin{array}{l} * \; u = sen(\omega t) \to du = \omega cos(\omega t) dt \\ dv = e^{-st} dt \to v = -\frac{e^{-st}}{s} \end{array}\right)$

$= \frac{\omega}{s}\mathcal{L}[cos(\omega t)]$ //

$\mathcal{L}[cos(\omega t)] = \int_0^\infty cos(\omega t) e^{-st} dt \overset{*}{=} -\frac{cos(\omega t) e^{-st}}{s}\Bigg|_0^\infty - \int_0^\infty \frac{\omega}{s} e^{-st} sen(\omega t) dt$

$\left(\begin{array}{l} * \; u = cos(\omega t) \to du = -\omega sen(\omega t) dt \\ dv = e^{-st} dt \to v = -\frac{e^{-st}}{s} \end{array}\right)$

$= \frac{1}{s} - \frac{\omega}{s}\mathcal{L}[sen(\omega t)]$

como $\mathcal{L}[sen(\omega t)] = \frac{\omega}{s}\mathcal{L}[cos(\omega t)]$ e $\mathcal{L}[cos(\omega t)] = \frac{1}{s} - \frac{\omega}{s}\mathcal{L}[sen(\omega t)]$:

$\mathcal{L}[sen(\omega t)] = \frac{\omega}{\omega^2 + s^2}$ //

$\mathcal{L}[cos(\omega t)] = \frac{s}{\omega^2 + s^2}$ //

spiral

10. $\mathcal{L}[f*g] = \int_0^\infty (f*g)\, e^{-st}\, dt = \int_0^\infty \left(\int_0^\infty f(t-\tau)\, g(\tau)\, d\tau \right) e^{-st}\, dt$

$$= \int_0^\infty \int_0^\infty f(t-\tau)\, g(\tau)\, e^{-st}\, dt\, d\tau \overset{*}{=} \int_0^\infty \int_0^\infty f(\beta)\, g(\tau)\, e^{-s(\beta+\tau)}\, d\beta\, d\tau$$

$(*\ \beta = t - \tau \rightarrow d\beta = d\tau)$

$$= \int_0^\infty f(\beta)\, e^{-s\beta}\, d\beta \cdot \int_0^\infty g(\tau)\, e^{-s\tau}\, d\tau = F(s) \cdot G(s)$$

11. $y(t) = \frac{7}{3} e^{-t} + \frac{3}{2} e^{-2t} - \frac{1}{6} e^{-4t}$

$Y(s) = \mathcal{L}[y(t)]$

$$Y(s) = \frac{7}{3} \frac{1}{1+s} + \frac{3}{2} \frac{1}{2+s} - \frac{1}{6} \frac{1}{4+s}$$

$u(t) = \delta(t) \rightarrow \mathcal{L}[u(t)] = \mathcal{L}[\delta(t)] \rightarrow U(s) = 1$

$$G(s) = \frac{Y(s)}{U(s)} = Y(s) \rightarrow G(s) = \frac{7}{3(1+s)} + \frac{3}{2(2+s)} - \frac{1}{6(4+s)}$$

12. $y(t) = 1 - \frac{7}{3} e^{-t} + \frac{3}{2} e^{-2t} - \frac{1}{6} e^{-4t}$

$Y(s) = \mathcal{L}[y(t)]$

$$Y(s) = \frac{1}{s} - \frac{7}{3} \cdot \frac{1}{s+1} + \frac{3}{2} \cdot \frac{1}{s+2} - \frac{1}{6} \cdot \frac{1}{s+4}$$

$$G(s) = \frac{Y(s)}{U(s)} = s\,Y(s) \rightarrow G(s) = 1 - \frac{7s}{3(s+1)} + \frac{3s}{2(s+2)} - \frac{s}{6(s+4)}$$