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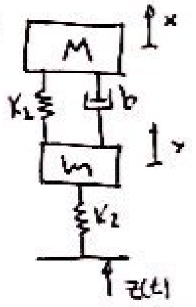
Ex. 2)



$M\ddot{x} + b(\dot{x} - \dot{y}) + K_1(x-y) = 0$

$m\ddot{y} + b(\dot{y} - \dot{x}) + K_1(y-x) + K_2(y-z) = 0$

$x_1 = x \qquad \dot{x}_1 = x_3$

$x_2 = y \qquad \dot{x}_2 = x_4$

$x_3 = \dot{x} \qquad \dot{x}_3 = [-b(x_3 - x_4) - K_1(x_1 - x_2)]/M$

$x_4 = \dot{y} \qquad \dot{x}_4 = [-b(x_4 - x_3) - K_1(x_2 - x_1) - K_2(x_2 - z)]/m$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-K_1}{M} & \frac{K_1}{M} & \frac{-b}{M} & \frac{b}{M} \\ \frac{K_1}{m} & \frac{-2K_2}{m} & \frac{b}{m} & \frac{-b}{m} \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_2}{m} \end{bmatrix}; C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; D = [0]$$

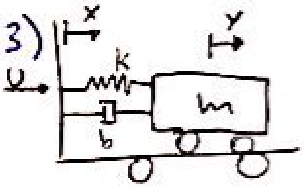
$X = [x \ y \ \dot{x} \ \dot{y}]; \quad Y = [x \ \dot{y}]$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = A.X + B.U \qquad Y = C.X$$

$m\ddot{y} = K(x-y) + b(\dot{x} - \dot{y})$

$M\ddot{x} = U - K(x-y) - b(\dot{x} - \dot{y})$

Ex. 3)



Novamente é usado espaço de estados, como acima

3.1) $Ax = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ x_4 \end{bmatrix}; \; BU = \begin{bmatrix} 0 \\ 1/m \end{bmatrix} U; \; CX = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ x_4 \end{bmatrix}$

3.2) $$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-K}{m} & \frac{K}{m} & \frac{-b}{m} & \frac{b}{m} \\ \frac{K}{m} & \frac{-K}{m} & \frac{b}{m} & \frac{-b}{m} \end{bmatrix}; \; B = \begin{bmatrix} 0 \\ 0 \\ 1/m \\ 0 \end{bmatrix} U; \; C = [0 \ 1 \ 0 \ 0]; \; D = [0]$$

Ex. 4) Foram obtidas as eqs. anteriormente.

$$\ddot{x}_1 = \frac{-(K_p + K_1)x_1}{m_1} - \frac{b_1 \dot{x}_1}{m_1} + \frac{K_p z(t)}{m_1} + \frac{K_1 x_G}{m_1} + \frac{K_1 b \theta}{m_1} + \frac{b_1 \dot{x}_G}{m_1} + \frac{b_1 b \theta}{m_1}$$

$$\ddot{x}_2 = \frac{-(K_p + K_2)x_2}{m_2} - \frac{b_2 \dot{x}_2}{m_2} + \frac{K_p z(t - \ell/v)}{m_2} + \frac{K_2 x_G}{m_2} - \frac{K_2 a \theta}{m_2} + \frac{b_2 \dot{x}_G}{m_2} - \frac{b_2 a \theta}{m_2}$$

$$\ddot{X}_G = \frac{-K_1 X_G}{M} - \frac{K_1 b\theta}{M} - \frac{b_1 \dot{X}_G}{M} - \frac{b_1 b\dot{\theta}}{M} - \frac{K_2 X_G}{M} + \frac{K_2 \partial\theta}{M} - \frac{b_2 \dot{X}_G}{M} + \frac{b_2 \partial\dot{\theta}}{M} + \frac{K_1 X_1}{M} + \frac{K_2 X_2}{M} + \frac{b_1 \dot{X}_1}{M} - \frac{b_2 \dot{X}_2}{M}$$

$$\ddot{\theta} = \frac{(K_2\partial - K_1 b)X_G}{J} + \frac{(b_2\partial - b_1 b)\dot{X}_G}{J} - \frac{(K_2\partial^2 + K_1 b^2)\theta}{J} - \frac{(b_2\partial^2 + b_1 b^2)\dot{\theta}}{J} + \frac{K_1 x_1 b}{J} + \frac{b_1 \dot{x}_1 b}{J} - \frac{K_2 x_2 \partial}{J} - \frac{b_2 \dot{x}_2 \partial}{J}$$

$X = [x_1 \; x_2 \; X_G \; \theta \; \dot{x}_1 \; \dot{x}_2 \; \dot{X}_G \; \dot{\theta}]^T$; $Y = [X_G \; \theta]^T$; $U = [z(t) \;\; z(t - \ell/v)]^T$

$\dot{X} = Ax + BU$; $Y = Cx + DU$ com:

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-(K_p+K_1)}{m_1} & 0 & \frac{K_1}{m_1} & \frac{K_1 b}{m_1} & \frac{-b_1}{m_1} & 0 & \frac{b_1}{m_1} & \frac{b_1 b}{m_1} \\ 0 & \frac{-(K_p+K_2)}{m_2} & \frac{K_2}{m_2} & \frac{-K_2\partial}{m_2} & 0 & \frac{-b_2}{m_2} & \frac{b_2}{m_2} & \frac{-b_2\partial}{m_2} \\ \frac{K_1}{M} & \frac{K_2}{M} & \frac{-(K_1+K_2)}{M} & \frac{(K_2\partial - K_1 b)}{M} & \frac{b_1}{M} & \frac{b_2}{M} & \frac{-(b_1+b_2)}{M} & \frac{(b_2\partial - b_1 b)}{M} \\ \frac{K_1 b}{J} & \frac{-K_2\partial}{J} & \frac{(K_2\partial - K_1 b)}{J} & \frac{-(K_2\partial^2 + K_1 b^2)}{J} & \frac{b_1 b}{J} & \frac{-b_2\partial}{J} & \frac{(b_2\partial - b_1 b)}{J} & \frac{-(b_2\partial^2 - b_1 b^2)}{J} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{K_p}{m_1} & 0 \\ 0 & \frac{K_p}{m_2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Ex.5) Eqn Linearizadas

$$\begin{cases}(M+m)\ddot{x} + ml\ddot{\theta} = U \\ J\ddot{\theta} = lmg\,\theta - ml\ddot{x}\end{cases} \Rightarrow \ddot{x} = \left(\frac{-m^2 l^2}{MJ + mJ - m^2 l^2}\right)\theta + \frac{U}{M+m-\frac{m^2 l^2}{J}}$$

$$\Downarrow$$

$$\ddot{\theta} = \left(\frac{gml(M+m)}{J(M+m) - m^2 l^2}\right)\theta - \left(\frac{gml}{J(M+m)-m^2 l^2}\right)U$$

- Definindo: $X = [x \ \theta \ \dot{x} \ \dot{\theta}]$; $Y = [x \ \theta]$; $U = U$

- sistema: $\dot{X} = AX + BU$; $Y = CX$ com:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \left(\frac{-m^2 l^2}{(M+m)J - m^2 l^2}\right) & 0 & 0 \\ 0 & \left(\frac{gml(M+m)}{J(M+m)-m^2 l^2}\right) & 0 & 0 \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ 0 \\ \left(\frac{1}{M+m-\frac{m^2 l^2}{J}}\right) \\ \left(\frac{-gml}{J(M+m)-m^2 l^2}\right) \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} ; \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Ex.6) $m\ddot{x} = mg - Ki^2/x^2$

$$L\frac{\partial i}{\partial t} + Ri = V$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = g - \left(\frac{K x_3^2}{m x_1^2}\right)$$

$$\dot{x}_3 = \frac{-R}{L}x_3 + \frac{1}{L}V$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2Kx_3^2}{mx_1^3} & 0 & \frac{-2Kx_3}{mx_1^2} \\ 0 & 0 & -R/L \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix}$$

Ex.7) a) $L_a \frac{\partial i_a}{\partial t} + R_a i_a = e_a - K_b(t)\,\omega(t) \rightarrow L\frac{\partial i}{\partial t} + Ri = V_a(t) - K\Omega_1$

$$J\ddot{\theta}_1 + B\dot{\theta}_1 = Ki_a - T_N \longrightarrow J_m\ddot{\theta}_1 + B_m\Omega_1 = Ki - 2|\theta_1 - \theta_2|(\theta_1-\theta_2)$$

$$m\ddot{x} = T_N - R - Kx_1 - F_{at} \longrightarrow m\ddot{x}_1 = 2R|\theta_1-\theta_2|(\theta_1-\theta_2) - Kx_1$$

b)
$$\left.\begin{aligned} x_1 &= x_1 \\ x_2 &= \theta_1 \\ x_3 &= \theta_2 \\ x_4 &= \int i\,dt \\ x_5 &= \dot{x}_1 \\ x_6 &= \dot{\theta}_1 \\ x_7 &= \dot{\theta}_2 \\ x_8 &= i \end{aligned}\right\} \quad \begin{aligned} \dot{x}_1 &= x_5 \\ \dot{x}_2 &= x_6 \\ \dot{x}_3 &= x_7 \\ \dot{x}_4 &= x_8 \\ \dot{x}_5 &= [2R|x_2-x_3|(x_2-x_3)\cdot K x_1]/m \\ \dot{x}_6 &= [K x_8 - 2|x_2-x_3|(x_2-x_3) - B_m x_6]/J_m \\ \dot{x}_7 &= 0 \\ \dot{x}_8 &= [V_a(t) - K x_6 - R x_8]/L \end{aligned}$$

Ex. 8) $H_x = J_x \cdot \omega_x \rightarrow$ momento ang.

a) $$\dot{\omega}_x + H_x\left(\frac{1}{J_D} - \frac{1}{J_x}\right)\cdot \omega_z = \frac{\tau_x}{J_d}$$

$$\dot{\omega}_z + H_x\left(\frac{1}{J_D} - \frac{1}{J_x}\right)\cdot \omega_x = 0$$

• $\tau_x = -\theta_x \cdot L \cdot 2K - B\omega_x$ e $J_y = J_z = J_D$

• $$\dot{\omega}_x + H_x\left(\frac{1}{J_D} - \frac{1}{J_x}\right)\omega_z = \frac{-2LK\,\theta_x}{J_D} - \frac{B\omega_x}{J_D}$$

Defini-se: $x = [\theta_x \ \dot{\theta}_x]$; $\dot{\theta}_x = \omega_x$; $\omega_z = u$

$$\ddot{\theta}_x = -\frac{2LK}{J_D}\theta_x - \frac{B\dot{\theta}_x}{J_D} - H_x\left(\frac{1}{J_D} - \frac{1}{J_x}\right)u$$

$$\underline{\dot{\theta}_x = \dot{\theta}_x}$$

b) $$\begin{bmatrix} \dot{\theta}_x \\ \ddot{\theta}_x \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-2LK}{J_D} & \frac{-B}{J_D} \end{bmatrix} \begin{bmatrix} \theta_x \\ \dot{\theta}_x \end{bmatrix} + \begin{bmatrix} 0 \\ -H_x\left(\frac{1}{J_D} - \frac{1}{J_x}\right) \end{bmatrix} \omega_z$$