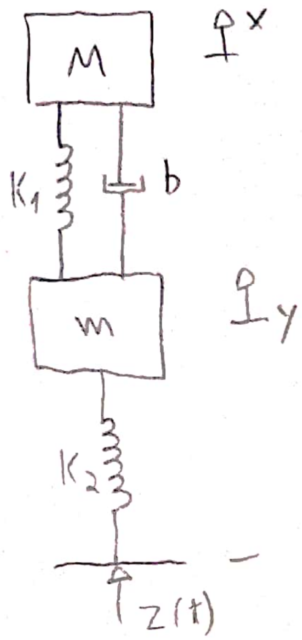
Leonardo Isaac de Oliveira - 10706131

PME3380 - Modelagem de Sistemas Dinâmicos

Exercícios Aula 06/10

2)

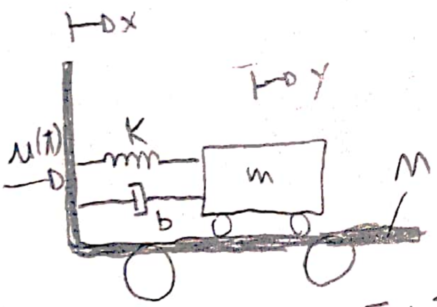


$$\Rightarrow \begin{cases} M\ddot{x} = -K_1(x-y) - b(\dot{x}-\dot{y}) \\ m\ddot{y} = K_1(x-y) + b(x-y) - K_2(y - z(t)) \end{cases}$$

$$\Rightarrow \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_1}{M} & -\frac{b}{M} & \frac{K_1}{M} & \frac{b}{M} \\ 0 & 0 & 0 & 1 \\ \frac{K_1}{m} & -\frac{b}{m} & \frac{-K_1-K_2}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_2}{m} \end{bmatrix} Z(t)$$

---

3)



$$\Rightarrow \begin{cases} M\ddot{x} = -K(x-y) - b(\dot{x}-\dot{y}) + u(t) \\ m\ddot{y} = K(x-y) + b(\dot{x}-\dot{y}) \end{cases}$$

$$\Rightarrow \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K}{M} & -\frac{b}{M} & \frac{K}{M} & \frac{b}{M} \\ 0 & 0 & 0 & 1 \\ \frac{K}{m} & \frac{b}{m} & -\frac{K}{m} & \frac{-b}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ 0 \end{bmatrix} u(t)$$

---

4) Das resoluções das aulas anteriores, temos:

$$\begin{cases} m_1\ddot{x}_1 = -K(x_1 - z) + K_1(x_G - x_1 + l\theta) + b_1(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) \\ m_2\ddot{x}_2 = -K(x_2 - z) + K_2(x_G - x_2 - l\theta) + b_2(\dot{x}_G + \dot{x}_2 - l\dot{\theta}) \\ M x_G = -K_1(x_G - x_1 + l\theta) - K_2(x_G - x_2 - l\theta) - b_1(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) - b_2(\dot{x}_G - \dot{x}_2 - l\dot{\theta}) \\ J_G\ddot{\theta} = -K_1 l(x_G - x_1 + l\theta) + K_2 l(x_G - x_2 - l\theta) - b_1 l(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) - b_2 l(x_G - \dot{x}_2 - l\theta) \end{cases}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_G \\ \dot{\theta} \\ \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_G \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{(K_1+K_1)}{m_1} & 0 & \frac{K_1}{m_1} & \frac{K_1 l}{m_1} & -\frac{b_1}{m_1} & 0 & \frac{b_1}{m_1} & \frac{b_1 l}{m_1} \\ 0 & -\frac{(K+K_2)}{m_2} & \frac{K_2}{m_2} & \frac{-K_2 l}{m_2} & 0 & \frac{-b_2}{m_2} & \frac{b_2}{m_2} & \frac{-b_2 l}{m_2} \\ \frac{K_1}{M} & \frac{K_2}{M} & -\frac{(K_1+K_2)}{M} & \frac{l(K_2-K_1)}{M} & \frac{b_1}{M} & \frac{b_2}{M} & \frac{-(b_1+b_2)}{M} & \frac{l(b_2-b_1)}{M} \\ \frac{K_1 l}{J_G} & -\frac{K_2 l}{J_G} & \frac{l(K_2-K_1)}{J_G} & \frac{l^2(K_1+K_2)}{J_G} & \frac{b_1 l}{J_G} & -\frac{b_2 l}{J_G} & \frac{l(b_2-b_1)}{J_G} & -\frac{l^2(b_1+b_2)}{J_G} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_G \\ \theta \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_G \\ \dot{\theta} \end{bmatrix}$$

---

5) $$\begin{cases} (M+m)\ddot{x} + ml\ddot{\theta} = u(t) \\ J\ddot{\theta} + ml\ddot{x} = mlg\theta \end{cases}$$

$$\Rightarrow \begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & a_{32} & 0 & 0 \\ 0 & a_{42} & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b_{31} \\ b_{41} \end{bmatrix} u(t)$$

onde: $a_{32} = \frac{-m^2 l^2}{J(M+m) - m^2 l^2}$ $\qquad b_{31} = \frac{1}{M+m-\frac{m^2 l^2}{J}}$

$a_{42} = \frac{gml(M+m)}{J(M+m) - m^2 l^2}$ $\qquad b_{41} = \frac{-gml}{J(M+m) - m^2 l^2}$

---

6) $$\begin{cases} m\ddot{x} = mg - \frac{KI^2}{x^2} \\ L\dot{I} + RI = V(t) \end{cases}$$

$$\Rightarrow \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{I} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2KI_0^2}{mx_0^3} & 0 & -\frac{2KI_0}{mx_0^2} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ I \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} V(t)$$