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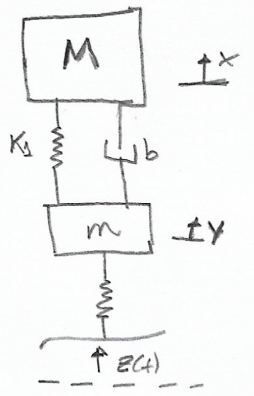
# PME3380 - Modelagem de Sistemas Dinâmicos

## Exercício da Aula do dia 01 e 06/10/2020

2)

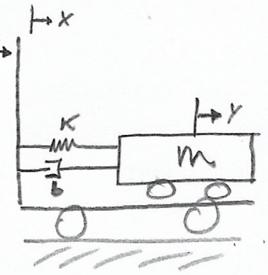


$$\begin{cases} M\ddot{x} = -b\cdot(\dot{x}-\dot{y}) - k_1\cdot(x-y) \\ m\ddot{y} = b\cdot(\dot{x}-\dot{y}) + k_1\cdot(x-y) - k_2\cdot(y-z(t)) \end{cases}$$

Representando no espaço de estados:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{k_1}{M} & \frac{k_1}{M} & -\frac{b}{M} & \frac{b}{M} \\ \frac{k_1}{m} & -\frac{(k_1+k_2)}{m} & \frac{b}{m} & -\frac{b}{m} \end{bmatrix} + z(t)\cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_2}{m} \end{bmatrix}$$

3)



$$\begin{cases} M\ddot{x} - b\cdot(\dot{y}-\dot{x}) - k\cdot(y-x) = u(t) \\ m\ddot{y} + b(\dot{y}-\dot{x}) + k\cdot(y-x) = 0 \end{cases}$$

Representando no espaço de estados:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{M} & \frac{k}{M} & -\frac{b}{M} & \frac{b}{M} \\ \frac{k}{m} & -\frac{k}{m} & \frac{b}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + u\cdot \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \\ 0 \end{bmatrix}$$

4) $$\begin{cases} m_1\ddot{x}_1 - b_1\cdot(\dot{x}_G + l\dot{\theta} - \dot{x}_1) + k(x_1 - z) - k_1\cdot(x_G + l\theta - x_1) = 0 \\ m_2\ddot{x}_2 - b_2\cdot(\dot{x}_G - l\dot{\theta} - \dot{x}_2) + k\cdot(x_2 - z) - k_2\cdot(x_G - l\theta - x_2) = 0 \\ M\ddot{x}_G + b_2\cdot(\dot{x}_G - l\dot{\theta} - \dot{x}_2) + b_1\cdot(\dot{x}_G + l\dot{\theta} - \dot{x}_1) + k_2\cdot(x_G - l\theta - x_2) + k_1\cdot(x_G + l\theta - x_1) = 0 \\ J_G\ddot{\theta} + b_2 l\cdot(\dot{x}_G - l\dot{\theta} - \dot{x}_2) + b_1 l(\dot{x}_G + l\dot{\theta} - \dot{x}_1) + k_2 l(x_G - l\theta - x_2) + k_1 l(x_G + l\theta - x_1) = 0 \end{cases}$$

→ Espaço de Estados: $\boxed{\dot{x} = Ax + Bu}$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-(k+k_1)}{m_1} & 0 & \frac{k_1}{m_1} & \frac{k_1 l}{m_1} & \frac{-b_1}{m_1} & 0 & \frac{b_1}{m_1} & \frac{b_1 l}{m_1} \\ 0 & \frac{-(k+k_2)}{m_2} & \frac{k_2}{m_2} & \frac{-k_2 l}{m_2} & 0 & \frac{-b_2}{m_2} & \frac{b_2}{m_2} & \frac{-b_2 l}{m_2} \\ \frac{k_1}{M} & \frac{k_2}{M} & -\frac{(k_1+k_2)}{M} & \frac{(k_2-k_1)l}{M} & \frac{b_1}{M} & \frac{b_2}{M} & \frac{-(b_1+b_2)}{M} & \frac{(b_2-b_1)l}{M} \\ \frac{k_1 l}{J} & -\frac{k_2 l}{J} & \frac{(k_2-k_1)l}{J} & \frac{(k_2+k_1)\cdot -l^2}{J} & \frac{b_1 l}{J} & \frac{-b_2 l}{J} & \frac{(b_2-b_1)l}{J} & \frac{(b_2-b_1)\cdot -l^2}{J} \end{bmatrix}$$

5) $$\begin{cases} (M+m)\ddot{x} + ml\ddot{\theta} = u \\ J\ddot{\theta} + ml\ddot{x} - mgl\theta = 0 \end{cases}$$

→ Espaço de Estados: $\boxed{\dot{x} = Ax + Bu}$

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-m^2l^2}{J\cdot(M+m) - m^2l^2} & 0 & 0 \\ 0 & \frac{mgl(M+m)}{J(M+m) - m^2l^2} & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + u \begin{bmatrix} 0 \\ 0 \\ \frac{1}{J(M+m) - m^2l^2} \\ -\frac{mgl}{J(M+m) - m^2l^2} \end{bmatrix}$$

6) $\begin{cases} m(\ddot{x}-g) + \dfrac{KI^2}{x^2} = 0 \\ L\dot{I} + RI = V \end{cases}$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{I} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2KI_0^2}{mx_0^3} & 0 & \frac{-2KI_0}{mx_0^2} \\ 0 & 0 & \frac{-R}{L} \end{bmatrix} + u \cdot \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}$$

8) $J_x\ddot{\theta}_x + 2b\dot{\theta}_x + kl\,\theta_x = J_w\,\theta_z$

$\rightarrow \dot{\theta} = A\theta + Bu$

$$\begin{bmatrix} \dot{\theta}_x \\ \ddot{\theta}_x \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{kl}{-J_x} & \frac{-2b}{J_x} \end{bmatrix} \cdot \begin{bmatrix} \theta_x \\ \dot{\theta}_x \end{bmatrix} + u \begin{bmatrix} 0 \\ -\frac{I}{J_x} \end{bmatrix}$$