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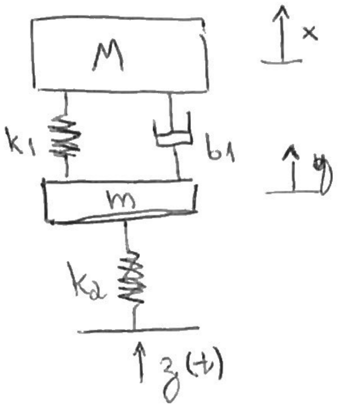
PME 3380 - Modelagem

$\boxed{\text{Ex 2}}$

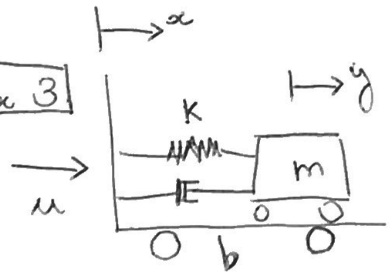


$$\begin{cases} M\ddot{x} + k_1(x-y) + b(\dot{x}-\dot{y}) = 0 \\ m\ddot{y} - k_1(x-y) - b(\dot{x}-\dot{y}) + k_2(y-z) = 0 \end{cases}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_1/M & k_1/M & -b/M & b/M \\ k_1/m & (k_1+k_2)/m & b/m & -b/m \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ k_2/m \end{bmatrix} z(t)$$

$$\dot{u} = Au + Bz \longrightarrow u = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}$$

$\boxed{\text{Ex 3}}$



$$\begin{cases} m\ddot{y} + k(y-x) + b(\dot{y}-\dot{x}) = 0 \\ M\ddot{x} - k(y-x) - b(\dot{y}-\dot{x}) = u \end{cases}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K/M & K/M & -b/M & b/M \\ K/M & -K/M & b/M & -b/M \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/M \\ 0 \end{bmatrix} u(t)$$

$$\dot{z} = Az + Bu \; ; \; z = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} \longleftarrow$$

$\boxed{\text{Ex 4}}$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_G \\ \theta \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_G \\ \dot{\theta} \end{bmatrix}$$

$$\begin{cases} m_1\ddot{x}_1 + k(x_1 - z) - k_1(x_G - x_1 + l\theta) - b_1(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) \\ m_2\ddot{x}_2 + k(x_2 - z) - k_2(x_G - x_2 - l\theta) - b_2(\dot{x}_G - \dot{x}_2 - l\dot{\theta}) \\ M\ddot{x}_G + k_1(x_G - x_1 + l\theta) + k_2(x_G - x_2 - l\theta) + b_1(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) + b_2(\dot{x}_G - \dot{x}_2 + l\dot{\theta}) \\ J_G\ddot{\theta} + k_1 l(x_G - x_1 + l\theta) - k_2 l(x_G - x_2 - l\theta) + b_1 l(\dot{x}_G - \dot{x}_1 + l\theta) + b_2 l(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) \end{cases}$$

$\rightarrow$ <u>linearizadas</u>

Com isso, $\dot{x} = Ax + Bu$, com $u = [z(t), z(t\infty)]$:

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-(k_1+k_2)}{m_1} & 0 & k_1/m_1 & \frac{k_1 l}{m_1} & \frac{-k_1}{m_1} & 0 & b_1/m_1 & b_1 l/m_1 \\ 0 & \frac{-(k_1+k_2)}{m_2} & k_2/m_2 & \frac{-k_2 l}{m_2} & 0 & -b_2/m_2 & b_2/m_2 & -b_2 l/m_2 \\ k_1/m & k_2/M & -(k_1+k_2)/M & l(k_2-k_1)/M & b_1/M & b_2/m_2 & -(b_1+b_2)/M & l(b_2-b_1)/M \\ k_1 l/J & -k_2 l/J & l(k_2-k_1)/J & \frac{l^2(k_2+k_1)}{J} & \frac{b_1 l}{J} & \frac{b_2 l}{J} & l(b_2-b_1)/J & l^2(b_2-b_1)/J \end{bmatrix}$$

$\boxed{\text{Ex 5}}$

$$\begin{cases} (M+m)\ddot{x} + ml\ddot{\theta} = u \\ J\ddot{\theta} + ml\ddot{x} - mlg\theta = 0 \end{cases}$$

$\rightarrow$ equações linearizadas

$$x = \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} \qquad \boxed{\dot{x} = Ax + Bu}$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-m^2 l^2}{J(M+m) - m^2 l^2} & 0 & 0 \\ 0 & \frac{g m l (M+m)}{J(M+m) + m^2 l^2} & 0 & 0 \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m + M - \frac{m^2 l^2}{J}} \\ \frac{-gml}{J(m+M) - m^2 l^2} \end{bmatrix}$$

(6)

$$\begin{cases} m\ddot{x} = mg - \dfrac{kI^2}{x^2} \\ L\dot{I} + RI = V \end{cases}$$

$$X = \begin{bmatrix} x \\ \dot{x} \\ I \end{bmatrix} \quad ; \quad \dot{X} = AX + uB \quad ; \qquad u = V$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \dfrac{2kI_0^2}{m x_0^3} & 0 & \dfrac{-2kI_0}{m x_0^2} \\ 0 & 0 & \dfrac{-R}{L} \end{bmatrix} \quad ; \quad B = \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{L} \end{bmatrix}$$