Escola Politecnica da USP

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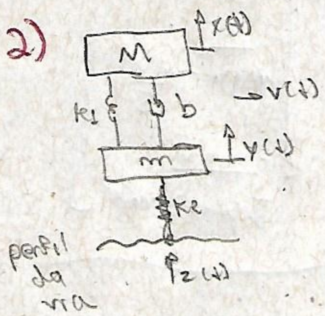
PME 3380 - Modelagem de Sistemas Dinâmicos - Ex de 01 e 06/10/2020

2)

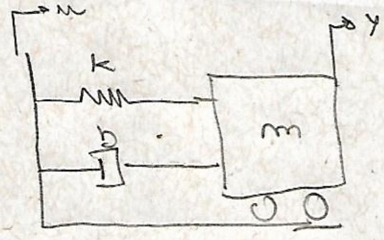


$$\begin{cases} M\ddot{x} + k_1(x-y) + b(\dot{x}-\dot{y}) = 0 \\ m\ddot{y} - k_1(x-y) - b(\dot{x}-\dot{y}) + k_2(y-z) = 0 \end{cases}$$ Equacoes Diferenciais

Espaco de Estados:

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1}{M} & \frac{k_1}{M} & -\frac{b}{M} & \frac{b}{M} \\ \frac{k_1}{m} & -\frac{k_1+k_2}{m} & \frac{b}{m} & -\frac{b}{m} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k}{m} \end{bmatrix} z = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix}$$

3)



Equacoes Diferenciais:

$$\begin{cases} M\ddot{x} - k(y-x) - b(\dot{y}-\dot{x}) = u \\ m\ddot{y} + b(\dot{y}-\dot{x}) + k(y-x) = 0 \end{cases}$$

Espaco de Estados

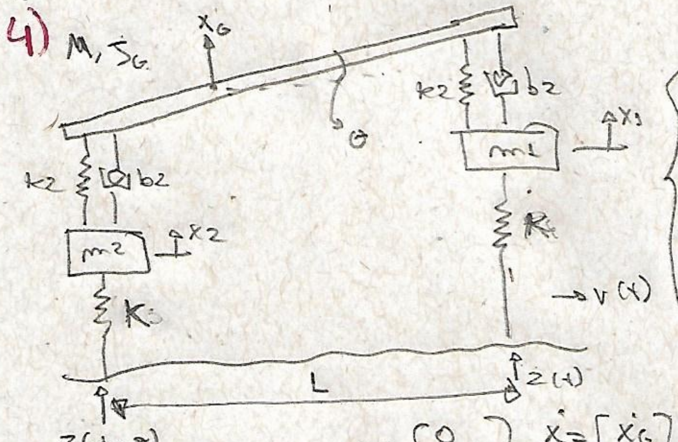
$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k/M & k/M & -b/M & b/M \\ k/M & -k/M & b/M & -b/M \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/M \\ 0 \end{bmatrix} u = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix}$$

$\dot{x} = Ax + Bu$

$y = Cx + Du$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} +$$

4)



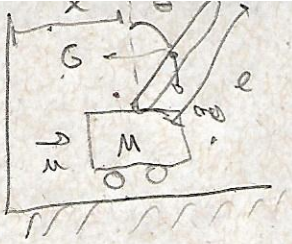
Equacoes Diferenciais:

$$\begin{cases} m_1\ddot{x}_1 + k(x_1 - z) + k_1(x_G - x_1 + l\theta) - b_1(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) = \\ m_2\ddot{x}_2 + k(x_2 - z) - k_2(x_G - x_2 + l\theta) - b_2(\dot{x}_G - x_2 - l\dot{\theta}) = \\ M\ddot{x}_G + k_1(x_G - x_1 + l\theta) + b_2(\dot{x}_G + \dot{x}_2 - l\dot{\theta}) + k_2(x_G - x_2 - l\dot{\theta} \\ J_G\ddot{\theta} + k_1 l(x_G - x_1 + l\theta) - k_2 l(x_G - x_2 - l\theta) + \\ \quad + b_1 l(\dot{x}_G - \dot{x}_1 + l\dot{\theta}) + b_2 l(\dot{x}_G - \dot{x}_2 - l\dot{\theta}) = 0 \end{cases}$$

Espaco de Estados:

$\dot{x} = Ax + Bu$

$$u = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ z(t) \end{bmatrix} \quad \dot{x} = \begin{bmatrix} \dot{x}_G \\ \dot{\theta} \\ \dot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_G \\ \ddot{\theta} \\ \ddot{x}_1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ k_1/m_1 & 0 \end{bmatrix} \quad A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{(k+k_1)}{m_1} & 0 & k_1/m_1 & \frac{k_1 l_1}{m_1} & -\frac{b_1}{m_1} & 0 & \frac{b_1}{m_1} \\ 0 & -\frac{(k+k_2)}{m_2} & k_2/m_2 & 0 & -\frac{b_2}{m_2} & \frac{b_2}{m_2} & -b \\ k_1/M & k_2/M & -\frac{(k_1+k_2)}{M} & \frac{b_1}{M} & \frac{b_2}{M} & & \end{bmatrix}$$

5) 

Equações Diferenciais:

$$\begin{cases} (M+m)\ddot{x} + ml\ddot{\theta} = u \\ J\ddot{\theta} + ml\ddot{x} - mg\dot{\theta} = 0 \end{cases}$$

$$\dot{x} = Ax + Bu, \quad x = \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix}, \quad u = u, \quad A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-m^2l^2}{J(M+m)-m^2l^2} & 0 & 0 \\ 0 & \frac{gml(M+m)}{J(M+m)-m^2l^2} & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M+m-\frac{m^2l^2}{J}} \\ \frac{-gml}{J(M+m)-m^2l^2} \end{bmatrix}$$

6) $$\dot{x} = Ax + Bu, \quad A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2kS_i^2}{mx_0^2} & 0 & \frac{-2kS}{mx_0^2} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix}, \quad u = V \quad e \quad x = \begin{bmatrix} x \\ \dot{x} \\ I \end{bmatrix}.$$

7)