Vitória Menino Campos nUSP: 10874175

Exercícios aula 01 e 06/10

Exercício 2:

$$M: \quad M\ddot{x} + b(\dot{x}-\dot{y}) + K_1(x-y) = 0$$

$$m: \quad m\ddot{y} + b(\dot{y}-\dot{x}) + K_1(y-x) + K_2(y-z) = 0$$

$\rightarrow x = [x \;\; y \;\; \dot{x} \;\; \dot{y}]$

$\rightarrow y = [x \;\; \dot{y}]$

$\rightarrow z = u$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K_1/M & K_1/M & -b/M & b/M \\ K_1/m & \frac{-(K_1+K_2)}{m} & b/m & -b/m \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ K_2/m \end{bmatrix}}_{B} \cdot u = \dot{X}$$

$$\begin{bmatrix} x \\ \dot{y} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{C} \cdot \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} = Y \qquad D = [0]$$

Exercício 3:

3.1:

$$m\ddot{y} + b(\dot{y}-\dot{x}) + K(y-x) = 0$$

$$M\ddot{x} + b(\dot{x}-\dot{y}) + K(x-y) = U$$

$\rightarrow x = [x \;\; y \;\; \dot{x} \;\; \dot{y}]$

$$A_x = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$$

$$B_u = \begin{bmatrix} 0 \\ 1/m \end{bmatrix} \cdot u \qquad C_x = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$$

3.2

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k/M & k/M & -b/M & b/M \\ k/m & -k/m & b/m & -b/m \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1/M \\ 0 \end{bmatrix} \cdot U \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Exercício 5

$$\begin{cases} 4/3\, m l^2 \ddot{\theta} + m \ddot{x}\, l \cos\theta - m g l \sin\theta = 0 \\ (M+m)\ddot{x} + m l \ddot{\theta} \cos\theta - m l \dot{\theta}^2 \sin\theta = u \end{cases}$$

$$\begin{cases} \ddot{x} = (m l \dot{\theta}^2 \sin\theta - m l \ddot{\theta} \cos\theta) / (M+m) \\ \ddot{\theta} = 4(m g l \sin\theta - m \ddot{x} l \cos\theta) / 3 m l^2 \end{cases}$$

$$\rightarrow \ddot{x} = (u + m l \dot{\theta}^2 \sin\theta - 3/4\, m g \cos\theta \sin\theta) / (M + m - 3/4\, m \cos^2\theta)$$

$$\rightarrow \ddot{\theta} \left[\frac{-m^2 l^2 \cos^2\theta}{M+m} + \frac{4 m l^2}{3} \right] + \frac{m^2 l^2 \dot{\theta}^2 \cos\theta \sin\theta}{M+m} - m g l \sin\theta = 0$$

$$x = [x \quad \theta \quad \dot{x} \quad \dot{\theta}]$$

$$\ddot{x} = \frac{U + ml\dot{\theta}^2 \sin\theta - 3/8\, mg \sin 2\theta}{M + m - 3/4\, m \cos\theta}$$

$$\ddot{\theta} = \frac{mgl \sin\theta - m^2 l^2 \sin 2\theta / 2(M+m)}{4/3\, ml^2 - m^2 l^2 \cos^2\theta / M+m}$$

Exercício 6:

- Não linear:

$$\begin{cases} m\ddot{x} = mg - Ki^2/x^2 \\ L\dot{i} + Ri = V \end{cases}$$

$$w(t) = [x \quad \dot{x} \quad i] = [w_1 \quad w_2 \quad w_3]$$

$$\begin{cases} \dot{w}_1 = w_2 \\ \dot{w}_2 = g - K w_3^2 / m w_1^2 \\ \dot{w}_3 = V(t)/L - R/L \cdot w_3 \end{cases}$$

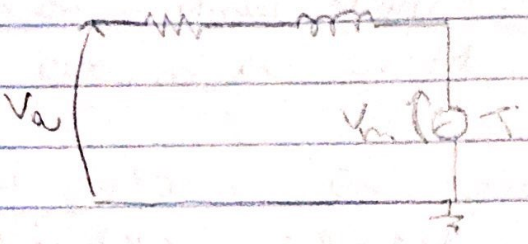
- Depois da linearização

$$\begin{bmatrix} \delta\dot{x} \\ \delta\ddot{x} \\ \delta\dot{i} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 2K i^2_{eq}/m x^3_{eq} & 0 & -2K i_{eq}/m x^2_{eq} \\ 0 & 0 & -R/L \end{bmatrix} \begin{bmatrix} \delta x \\ \delta\dot{x} \\ \delta i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix} \delta V(t)$$

Exercício 7

a.

Motor:



• Lei das malhas: $L\dot{i} + Ri + K_b\dot{\theta}_1 = V_a$

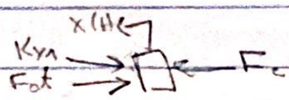
Disco:

• TMQM: $J_m\ddot{\theta}_1 + 2K_s(\theta_1-\theta_2)|\theta_1-\theta_2| + B_m\dot{\theta}_1 = K_a i$

Pinhão:

• TMQM: $J_p.\ddot{\theta}_2 = T_{NLK} - F_c R$

Carro:



• TMB: $\left(m + \frac{J_p}{R^2}\right)\ddot{x}_1 + 2K\dot{x}_1^3 + Kx_1 = \frac{2K_s(\theta_1 - x_1/R)|\theta_1 - x_1/R|}{R}$

b. Reescrevendo como variáveis de estado:

$x = [x\ \dot{x}\ \theta\ \dot{\theta}\ i]$

$\dot{i} = 1/L\,(V_a - K_b\dot{\theta} - Ri)$

$\ddot{\theta} = 1/J_m\,(K_a i - 2K_s(\theta_1 - x/R) - B_m\dot{\theta}_1)$

$\ddot{x} = R^2/(mR^2 + J_P).\,(2K_s(\theta_1 - x_1/R)/R - 2\dot{x}^3 - K_x)$

Exercício 8:

- Centro do disco como polo:

$$J_x \ddot{\theta}_x = -2B\dot{\theta}_x - 2Kd\theta_x - J\omega\theta_z$$

$$x = [\theta_x \ \dot{\theta}_x]$$

$$\begin{bmatrix} \dot{\theta}_x \\ \ddot{\theta}_x \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2Kd/J_x & -2B/J_x \end{bmatrix} \begin{bmatrix} \theta_x \\ \dot{\theta}_x \end{bmatrix} \cdot u$$

$$y(t) = [1 \ 0] \begin{bmatrix} \theta_x \\ \dot{\theta}_x \end{bmatrix}$$