Modelagem 6/10

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2. $\begin{cases} m\ddot{y} = -k_2(y-z) - k_1(y-x) - b(\dot{y}-\dot{x}) \\ M\ddot{x} = -k_1(x-y) - b(\dot{x}-\dot{y}) \end{cases}$ $u = z(t)$

$\Downarrow$

$x_1 = x$

$x_2 = \dot{x}$

$x_3 = y$ $\Rightarrow$ $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{1}{M}\left[-k_1(x_1-x_3) - b(x_2-x_4)\right] \\ x_4 \\ \frac{1}{m}\left[-k_2(x_3-u) - k_1(x_3-x_1) - b(x_4-x_2)\right] \end{bmatrix}$

$x_4 = \dot{y}$

Espaço de Estados (E.E.)

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix}}_{[\dot{X}]} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{M} & -\frac{b}{M} & \frac{k_1}{M} & \frac{b}{M} \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m} & \frac{b}{m} & -\frac{(k_1+k_2)}{m} & -\frac{b}{m} \end{bmatrix}}_{[A]} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{[X]} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_2}{m} \end{bmatrix}}_{[B]} u$$

3.1. massa correia desprezível.

$$\rightarrow m\ddot{y} = -b\dot{y} - ky + b\dot{x} + kx$$

$$\begin{cases} m\ddot{y} + b\dot{y} + ky = b\dot{x} + kx \\ b\dot{x} + kx = ky + b\dot{y} + u \end{cases} \Rightarrow \boxed{m\ddot{y} = u} \rightarrow b\dot{x} = -kx + ky + b\dot{y} + u$$

$$\begin{aligned} x_1 &= y \\ x_2 &= \dot{y} \\ x_3 &= x \\ x_4 &= \dot{x} \end{aligned} \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{1}{m}[b(x_4 - x_2) + k(x_3 - x_1)] \\ x_4 \\ \ \end{bmatrix} \quad \begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

$$\begin{aligned} x_1 &= y \\ x_2 &= \dot{y} \end{aligned} \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ u/m \end{bmatrix} \Rightarrow$$

$$\Rightarrow \underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}}_{[\dot{X}]} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{[A]} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{[X]} + \underbrace{\begin{bmatrix} 0 \\ 1/m \end{bmatrix}}_{[B]} u$$

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}_{[Y]} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{[C]} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{[D]} u$$

3.2. massa da correia ñ despres. (M)

$$\begin{cases} m\ddot{y} = b(\dot{x} - \dot{y}) + k(x - y) \\ M\ddot{x} = b(\dot{y} - \dot{x}) + k(y - x) + u \end{cases} \qquad \begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \\ x \\ \dot{x} \end{bmatrix} \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{1}{m}[b(x_4 - x_2) + k(x_3 - x_1)] \\ x_4 \\ \frac{1}{M}[b(x_2 - x_4) + k(x_1 - x_3) + u] \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & \frac{k}{m} & \frac{b}{m} \\ 0 & 0 & 0 & 1 \\ \frac{k}{M} & \frac{b}{M} & -\frac{k}{M} & -\frac{b}{M} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/M \end{bmatrix} u$$

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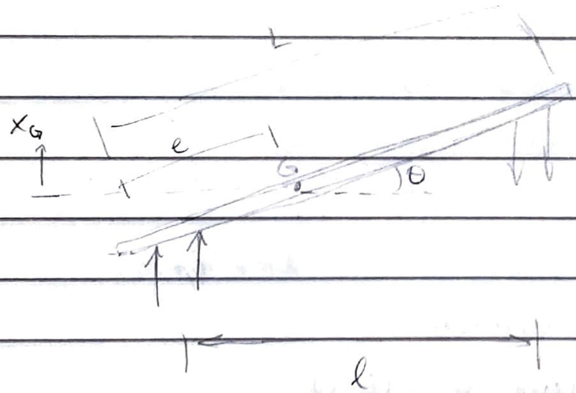
$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$

4-a) Grandes movimentos

$$\begin{cases} m_1 \ddot{x}_1 = k_1 [x_G + (L-e) \cdot \cos\theta - x_1] - k(x_1 - z) + b_1(\dot{x}_G - (L-e)\dot{\theta} \operatorname{sen}\theta - \dot{x}_1) \\ m_2 \ddot{x}_2 = k_2 [x_G - e\cos\theta - x_2] - k(x_2 - z) + b_2(\dot{x}_G + e\dot{\theta} \operatorname{sen}\theta - \dot{x}_2) \\ M\ddot{x}_G = -k_1[x_G + (L-e)\cos\theta - x_1] - k_2(x_G - e\cos\theta - x_2) - b_1[\dot{x}_G - (L-e)\dot{\theta}\operatorname{sen}\theta - \dot{x}_1] \\ \quad - b_2(\dot{x}_G + e\dot{\theta} \cdot \operatorname{sen}\theta - \dot{x}_2) \\ J_G \cdot \ddot{\theta} = -k_1 e \operatorname{sen}\theta [x_G + l - e\cos\theta - x_1] - k_2 e \operatorname{sen}\theta (x_G - e\cos\theta - x_2) - b_1(\dot{x}_G - l\dot{\theta} \cdot tg\theta + \\ \quad + e\dot{\theta} \operatorname{sen}\theta - \dot{x}_1)(e\operatorname{sen}\theta - l\,tg\theta) - b_2 e \operatorname{sen}\theta (\dot{x}_G + e\dot{\theta} \cdot \operatorname{sen}\theta - \dot{x}_2) \end{cases}$$



$u = z(t)$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ w_8 \end{bmatrix} = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \\ x_G \\ \dot{x}_G \\ \theta \\ \dot{\theta} \end{bmatrix} \Rightarrow \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \\ \dot{w}_4 \\ \dot{w}_5 \\ \dot{w}_6 \\ \dot{w}_7 \\ \dot{w}_8 \end{bmatrix} = \begin{bmatrix} w_2 \\ \frac{1}{m_1}\{k_1[w_5 + (L-e)\cos\theta - w_1] - k(w_1 - u) + b_1(w_6 - (L-e)\ldots \\ w_4 \\ \frac{1}{m_2}\{k_2[w_5 \ldots \\ w_6 \\ \ldots \\ w_8 \\ \ldots \end{bmatrix}$$

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$[\dot{X}]$ $[A]$

$$
\begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \\ \dot{w}_4 \\ \dot{w}_5 \\ \dot{w}_6 \\ \dot{w}_7 \\ \dot{w}_8 \end{bmatrix}
=
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{(k_1+k)}{m_1} & -\frac{b_1}{m_1} & 0 & 0 & \frac{k_1}{m_1} & \frac{b_1}{m_1} & \frac{k_1(L-e)}{m_1}\cos & -\frac{b_1(L-e)\operatorname{sen}(w_7)}{m_1} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{(k_2+k)}{m_2} & -\frac{b_2}{m_2} & \frac{k_2}{m_2} & \frac{b_2}{m_2} & \frac{-k_2 e\cos + b_2 e \operatorname{sen}}{m_2} & \frac{b_2 e}{m_2} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\frac{k_1}{M} & \frac{b_1}{M} & \frac{k_2}{M} & \frac{b_2}{M} & -\frac{(k_1+k_2)}{M} & -\frac{(b_1+b_2)}{M} & ? & ? \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
? & ? & ? & ? & ? & ? & ? & ?
\end{bmatrix}
\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ w_8 \end{bmatrix}
+
$$

$$
+ \begin{bmatrix} 0 \\ -\frac{k}{m_1} \\ 0 \\ -\frac{k}{m_2} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u
$$

$[B]$ $[X]$

b) Pequenos movimentos de theta (Linearização)

$\cos\theta = 1$, $\operatorname{sen}\theta = \theta$, $\operatorname{tg}\theta = \theta$, $\theta\cdot\dot{\theta} = 0$, $\dot{\theta}^2 = 0$

$$
\begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \\ \dot{w}_4 \\ \dot{w}_5 \\ \dot{w}_6 \\ \dot{w}_7 \\ \dot{w}_8 \end{bmatrix}
=
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{(k_1+k)}{m_1} & -\frac{b_1}{m_1} & 0 & 0 & \frac{k_1}{m_1} & \frac{b_1}{m_1} & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{(k_2+k)}{m_2} & -\frac{b_2}{m_2} & \frac{k_2}{m_2} & \frac{b_2}{m_2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\frac{k_1}{M} & \frac{b_1}{M} & \frac{k_2}{M} & \frac{b_2}{M} & -\frac{(k_1+k_2)}{M} & -\frac{(b_1+b_2)}{M} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
? & ? & ? & ? & ? & ? & ? & ?
\end{bmatrix}
\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ w_8 \end{bmatrix}
$$

5 Pêndulo invertido.

Não Linear

$$\begin{cases} (m+M)\ddot{x} + ml\left(\cos\theta \cdot \ddot{\theta} - \operatorname{sen}\theta \cdot \dot{\theta}^2\right) = u \\ \frac{4}{3} ml^2 \ddot{\theta} + ml\ddot{x} \cdot \cos\theta = mgl \cdot \operatorname{sen}\theta \end{cases}$$

Linearizando: $\theta \to$ pequeno

$$\begin{cases} (m+M)\ddot{x} + ml \cdot \ddot{\theta} = u & (1) \\ \frac{4}{3} ml^2 \ddot{\theta} + ml\ddot{x} = mgl\theta & (2) \end{cases}$$

Desacoplando.

$$\ddot{x} = \frac{u - ml\ddot{\theta}}{M+m} \Rightarrow \frac{4}{3} ml^2 \ddot{\theta} + ml\left(\frac{u - ml\ddot{\theta}}{m+M}\right) = mgl\theta \Rightarrow$$

$$\Rightarrow \left(\frac{4}{3} ml^2 - \frac{m^2 l^2}{m+M}\right)\ddot{\theta} - mgl\theta = -\frac{mlu}{m+M} \Rightarrow$$

$$\Rightarrow \boxed{\ddot{\theta} = \frac{\left(mlg\theta - \frac{mlu}{m+M}\right)}{\left(\frac{4}{3} ml^2 - \frac{m^2 l^2}{m+M}\right)}}$$

Substituindo em (1)

$$(m+M)\ddot{x} + ml\left[\frac{\left(\cancel{ml}g\theta - \frac{mlu}{m+M}\right)\cancel{ml}}{\left(\frac{4}{3}\cancel{ml}^2 - \frac{m^2 l^2}{m+M}\right)\cancel{ml}}\right] = u$$

$$\boxed{\ddot{x} = \frac{u}{m+M} - \left[\frac{\left(mlg\theta - \frac{mlu}{m+M}\right)}{\left(\frac{4}{3}(m+M)l - ml\right)}\right]}$$

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$\dot{X}$ A

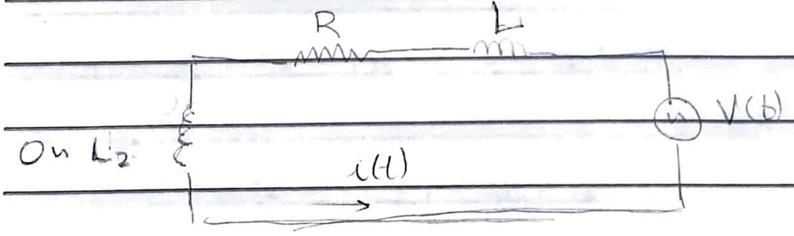
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} \Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -mgl/\left(\frac{4}{3}(m+M)l - ml\right) & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & g\theta/\left(\frac{4l}{3} - \frac{ml}{m+M}\right) & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

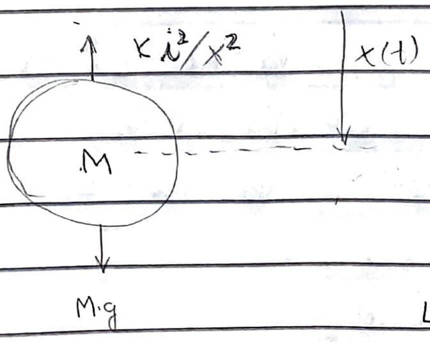
$$+ \underbrace{\begin{bmatrix} 0 \\ \left[\left(\frac{ml}{m+M}\right)/\left(\frac{4}{3}(m+M)l - ml\right)\right] + \frac{1}{(m+M)} \\ 0 \\ \left(\frac{1}{m+M}\right)/\left(\frac{4}{3}l - \frac{ml}{m+M}\right) \end{bmatrix}}_{B} u$$

Y

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \Rightarrow \begin{cases} y_1 = x_1 = x \\ y_2 = x_3 = \theta \end{cases}$$

6.





Esfera:

$$M\ddot{x} = Mg - \frac{Ki^2}{x^2} \Rightarrow M\ddot{x} + \frac{Ki^2}{x^2} = Mg$$

Malha.

$$LDi + Ri = V(t) \Rightarrow L\dot{i} + Ri = V$$

Não Linear $\Rightarrow$
$$\begin{cases} \ddot{x} = g - \frac{Ki^2}{Mx^2} \\ \dot{i} = \frac{V - Ri}{L} \end{cases}$$

FORONI

$$\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}x\\ \dot{x}\\ i\end{bmatrix} \Rightarrow \begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = g - \dfrac{Ki^2}{Mx^2}\\ \dot{x}_3 = \dfrac{V - Rx_3}{L}\end{cases}$$

Usando as variações:

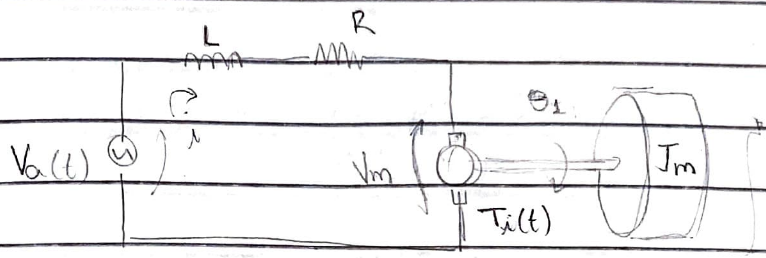
$$\begin{cases}\delta\dot{x}_1 = \delta x_2\\ \delta\dot{x}_2 = \dfrac{2Kx_{3eq}^2}{Mx_{1eq}^3}\,\delta x_1 + \left(\dfrac{-2Kx_{3eq}}{mx_{1eq}^2}\right)\delta x_3\\ \delta\dot{x}_3 = -\dfrac{R}{L}\,\delta x_3 + \dfrac{1}{L}\,\delta V\end{cases} \Rightarrow$$

$$\Rightarrow \underbrace{\begin{bmatrix}\delta\dot{x}_1\\ \delta\dot{x}_2\\ \delta\dot{x}_3\end{bmatrix}}_{\delta x} = \underbrace{\begin{bmatrix}0 & 1 & 0\\ \dfrac{2Kx_{3eq}^2}{Mx_{1eq}^2} & 0 & \dfrac{-2Kx_{3eq}}{Mx_{1eq}}\\ 0 & 0 & -R/L\end{bmatrix}}_{\delta A} + \overbrace{\begin{bmatrix}0\\ 0\\ 1/L\end{bmatrix}}^{B}\delta V$$

Saídas sendo x e i.

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \underbrace{\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}}_{C}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

7. Motor:



$$V_a - Ri - L\dot{i} - V_m = 0$$

$$V_m = K\cdot\dot{\theta}_1(t)$$

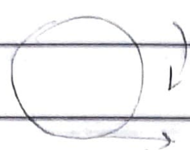
$$V_a - Ri - L\dot{i} - K\dot{\theta}_1 = 0$$

$$T = Ki(t) = K\dot{\theta}_1(t)$$

TMQM do motor:

$$J_m \ddot{\theta}_1 = K_i(t) - B_m \dot{\theta}_1 - 2|\theta_1 - \theta_2|(\theta_1 - \theta_2)$$

TMQM Pinhão:



$+2|\theta_1 - \theta_2|(\theta_1 - \theta_2)$ $\qquad J_p \ddot{\theta}_2 = 2(\theta_1 - \theta_2)|\theta_1 - \theta_2| - 2\dot{x}^3 \cdot R$

$$\vec{F} = 2\dot{x}^3$$

TMB Bloco:

$$m\ddot{x} = F - b\dot{x} - Kx \Rightarrow m\ddot{x} = 2\dot{x}^3 - b\dot{x} - Kx$$

Linearizando: $F = 2\dot{x}^3$ e $T = 2|\theta_1 - \theta_2|(\theta_1 - \theta_2)$

$$f(x) \cong f(\bar{x}) + \left.\frac{\partial f}{\partial x}\right|_{\bar{x}} (x - \bar{x})$$

$$F(x) \cong 2\bar{\dot{x}}^3 + 6\bar{\dot{x}}^2(\dot{x} - \bar{\dot{x}}) \Rightarrow F(x) \cong -4\bar{\dot{x}}^3 + 6\bar{\dot{x}}^2\dot{x}$$

$$T(x) \cong \begin{cases} 2(\bar{\theta}_1 - \bar{\theta}_2)^2 + 2(\bar{\theta}_1 - \bar{\theta}_2)(\theta_1 - \bar{\theta}_1) - 2(\bar{\theta}_1 - \bar{\theta}_2)(\theta_2 - \bar{\theta}_2) & p/\ \theta_1 > \theta_2 \\ -2(\bar{\theta}_1 - \bar{\theta}_2)^2 - 2(\bar{\theta}_1 - \bar{\theta}_2)(\theta_1 - \bar{\theta}_1) + 2(\bar{\theta}_1 - \bar{\theta}_2)(\theta_2 - \bar{\theta}_2) & p/\ \theta_1 < \theta_2 \\ 0 & p/\ \theta_1 = \theta_2 \end{cases}$$

Então:

$$\begin{cases} V_a - Ri - L\dot{i} - k\dot{\theta}_1 = 0 \\ J_m \ddot{\theta}_1 = Ki - B_m \dot{\theta}_1 - T(x) \\ J_p \ddot{\theta}_2 = T(x) - R4\bar{\dot{x}}^3 + R6\bar{\dot{x}}^2 \cdot \dot{x} \\ m\ddot{x} = 4R\bar{\dot{x}}^3 - 6\bar{\dot{x}}^2\dot{x} - Kx - b\dot{x} \end{cases}$$

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$$
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix}
=
\begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \\ x \\ \dot{x} \\ i \end{bmatrix}
\Rightarrow
\left|
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{J_m}\left(K x_7 - B_m x_2 - T(x)\right) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \left(T(x) - R4\bar{\dot{x}}^3 + R6\bar{\dot{x}}^2 \cdot x_6\right)\frac{1}{J_p} \\
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= \frac{1}{m}\left(4R\bar{\dot{x}}^3 - 6\bar{\dot{x}}^2 \cdot x_6 - K x_5 - b x_6\right) \\
\dot{x}_7 &= \frac{1}{L}\left(V_a - R x_7 - K x_2\right)
\end{aligned}
\right.
$$