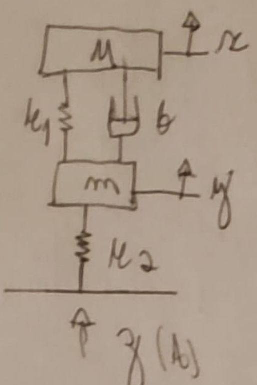
PME 3380 - Modelagem — Pedro L. G. Oliveira - 10335569

Exercícios aula ~~01/10~~ e 06/10

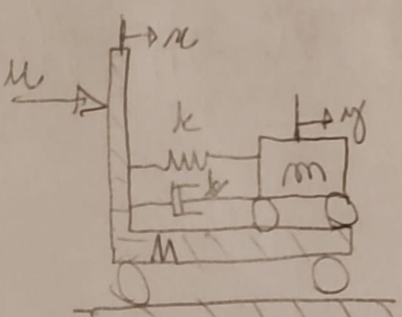
2)



$$\begin{cases} M: & M\ddot{x} + k_1(x-y) + b(\dot{x}-\dot{y}) = 0 \\ m: & m\ddot{y} + k_1(y-x) + b(\dot{y}-\dot{x}) = 0 \end{cases}$$

$$\dot{u} = A.u + B.y \Rightarrow \begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_1/M & k_1/M & -b/M & b/M \\ k_1/m & \frac{-k_1-k_2}{m} & b/m & -b/m \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ k_2/m \end{bmatrix} \cdot y(t)$$

3)



$$\begin{cases} M: & M\ddot{x} + k(x-y) + b(\dot{x}-\dot{y}) = u \\ m: & m\ddot{y} + k(y-x) + b(\dot{y}-\dot{x}) = 0 \end{cases}$$

$$\dot{y} = A.y + B.u \Rightarrow \begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k/M & k/M & -b/M & b/M \\ k/m & -k/m & b/m & -b/m \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/M \\ 0 \end{bmatrix} \cdot u$$

4) Definindo: $X = [x_1, x_2, x_g, \theta, \dot{x}_1, \dot{x}_2, \dot{x}_g, \dot{\theta}]$

Equações lineares:

.) $m_1\ddot{x}_1 + k(x_1 - y) + k_1(x_1 - x_g - l\theta) + b_1(\dot{x}_1 - \dot{x}_g - l\dot{\theta}$

.) $m_2\ddot{x}_2 + k(x_2 - y) + k_2(-x_g + x_2 + \theta l) + b_2(-\dot{x}_g + \dot{x}_2 + l\dot{\theta})$

.) $M\ddot{x}_g + k_1(x_g - x_1 + \theta l) + k_2(x_g - x_2 - l\theta) + b_1(\dot{x}_g - \dot{x}_1 + l\dot{\theta})$
$+ b_2(\dot{x}_g - \dot{x}_2 - l\dot{\theta})$

.) $J_g\ddot{\theta} + k_1 l(x_g - x_1 + l\theta) + k_2 l(-x_g + x_2 + l\theta)$
$+ b_1 l(\dot{x}_g - \dot{x}_1 + l\dot{\theta}) + b_2 l(\dot{x}_g - \dot{x}_2 - l\dot{\theta})$

Vetor de estados: $\dot{x} = A.X + B.U$, em que $U = [y(t), y(t-\alpha)]$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-k-k_1}{m_1} & 0 & k_1/m_1 & k_1.l/m_1 & -b_1/m_1 & 0 & b_1/m_1 & b_1.l/m_1 \\ 0 & \frac{-k_1-k_2}{m_2} & k_2/m_2 & \frac{-k_2.l}{m_2} & 0 & \frac{-b_2}{m_2} & b_2/m_2 & -b_2.l/m_2 \\ k_1/M & k_2/M & -k_1-k_2/M & l(k_2-k_1)/M & \frac{b_1}{M} & \frac{b_2}{M} & \frac{-b_1-b_2}{M} & \frac{l(b_2-b_1)}{M} \\ \frac{k_1.l}{J} & -k_2 l/J & \frac{l(k_2-k_1)}{J} & \frac{-l^2(k_2+k_1)}{J} & \frac{b_1.l}{J} & -\frac{b_2.l}{J} & \frac{l(b_2-b_1)}{J} & -\frac{l^2(b_2-b_1)}{J} \end{bmatrix}$$

5) Linearização:

$$\begin{cases} (M+m).\ddot{x} + m.l\ddot{\theta} = u \\ J.\ddot{\theta} + m\,l.\ddot{x} - m.l.g\,\theta = 0 \end{cases}$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-m.l^2}{J(m+M)-m^2.l^2} & 0 & 0 \\ 0 & \frac{g.m.l.(M+m)}{J(m+M)-m^2.l^2} & 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M+m-m^2.l^2/J} \\ \frac{-g.m.l}{J(m+M)-m^2.l^2} \end{bmatrix}; \quad x = \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix}$$

$$\underline{\dot{x} = A.x + B.u}$$

6) Equações do sistema: $\begin{cases} m.\ddot{x} = m.g - \dfrac{k i^2}{x^2} \\ L.\dot{i} + R.i = V \end{cases}$

Sendo: $x = \begin{bmatrix} x \\ \dot{x} \\ i \end{bmatrix}$; $y = [x]$; $u = V$

Sistema linear:

$$A = \begin{bmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial \dot{x}} & \frac{\partial \dot{x}}{\partial i} \\ \frac{\partial \ddot{x}}{\partial x} & \frac{\partial \ddot{x}}{\partial \dot{x}} & \frac{\partial \ddot{x}}{\partial i} \\ \frac{\partial \dot{i}}{\partial x} & \frac{\partial \dot{i}}{\partial \dot{x}} & \frac{\partial \dot{i}}{\partial i} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{-2k.i_0^2}{m.x_0^3} & 0 & \frac{-2k.i_0}{m.x_0^2} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix} ;$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix} \quad ; \quad \dot{x} = A.x + B.u$$

7) a) - Carro: $m.\ddot{x}_1 = 2|\theta_1 - \theta_2|.(\theta_1 - \theta_2).\frac{1}{R} - 2\dot{x}_1^3 - kx_1$

- Motor: $J_m.\dot{\omega}_1 = k.i - B_m\,\omega_1 - 2|\theta_1 - \theta_2|(\theta_1 - \theta_2)$

- Vínculo: $x_1 = \theta_2.R \Rightarrow \dot{x}_1 = \omega_2.R$

- Vetor de estados: $X = [i \ \theta \ x_1 \ \dot{\theta}_1 \ \dot{x}_1]^T$

b) Linearização:

$\cdot)\ f_1 = f(0) + \frac{\partial f}{\partial \theta_1}(\theta_1 - \theta_{10}) + \frac{\partial f}{\partial \theta_2}(\theta_2 - \theta_{20})$

$$\Rightarrow f_1 \cong 2 \cdot \left|\theta_{10} - \frac{x_{10}}{R}\right| \cdot \left(\theta_{10} - \frac{x_{10}}{R}\right) + 4\left|\theta_{10} - \frac{x_{10}}{R}\right| \cdot (\theta_1 - \theta_{10})$$
$$- 4 \cdot \left|\frac{x_{10}}{R} - \theta_{10}\right| (\theta_2 - \theta_{20})$$

Definindo: $\left(\theta_{10} - \frac{x_{10}}{R}\right) = \delta_0$; $(\theta_1 - \theta_{10}) = \theta_1$; $(\theta_2 - \theta_{20}) = \theta_2$

$\cdot)\ f_2 = 2 \cdot \dot{x}_i^3 = f_{eq} + \frac{\partial f}{\partial \dot{x}_1}\Big|_{eq} (\dot{x}_1 - \dot{x}_{10})$

$\Rightarrow f_2 = f_{20} + 6 \cdot \dot{x}_{10}^2 \cdot (\dot{x}_1 - \dot{x}_{10})$

Definindo: $(\dot{x}_1 - \dot{x}_{10}) = \dot{x}_1$

Assim, obtém-se:

$$i = \frac{V_a}{L} - \frac{k_1}{L} \cdot \dot{\theta}_1 - \frac{R_1}{L}$$

$$\ddot{\theta}_1 = \frac{k}{J_m} \cdot i - \frac{B_m}{J_m} \cdot \dot{\theta}_1 - \frac{4 \cdot \delta_0}{J_m}\left(\theta_1 - \frac{x_1}{R}\right) - \frac{\delta_0^2}{J_m}$$

$$\ddot{x}_1 = \frac{4 \cdot \delta_0}{m \cdot R}\left(\theta_1 - \frac{x_1}{R}\right) + \frac{\delta_0^2}{m \cdot R} - 6 \cdot x_{10}^2 \cdot \dot{x}_1 + f_{20} - k \cdot x_1$$

c) EE:

$$A = \begin{bmatrix} -R/L & 0 & 0 & -\frac{k_b}{L} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \frac{k}{J_m} & \frac{-4\delta_0}{J_m} & \frac{4\delta_0}{J_m . R} & \frac{-B_m}{J_m} & 0 \\ 0 & \frac{4\delta_0}{m.R} & \frac{-4\delta_0 - k}{m.R^2} & 0 & -6x_{10}^2 \end{bmatrix};$$

$$B = \begin{bmatrix} 1/L \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix};$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$