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Nome: Bruno Akira Oshiro   NUSP: 10771667

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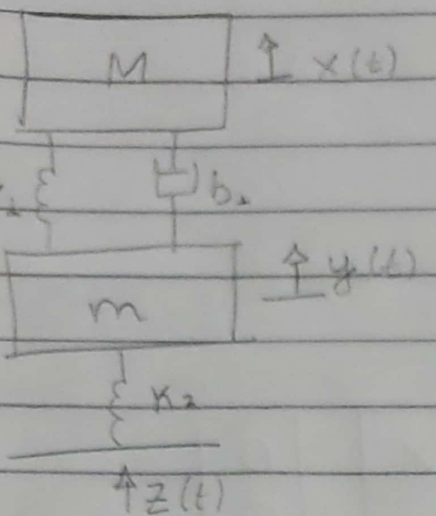

$$\begin{cases} M\ddot{x} + b_1\dot{x} - b_1 y + K_1 x - K_1 y = 0 \\ m\ddot{y} - b_1\dot{x} + b_1\dot{y} - K_1\dot{x} + (K_1+K_2)y = K_2 z \end{cases}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_1}{M} & +\frac{K_1}{M} & -\frac{b_1}{M} & \frac{b_1}{M} \\ \frac{K_1}{m} & -\frac{(K_1+K_2)}{m} & \frac{b_1}{m} & -\frac{b_2}{m} \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ K_2 \end{bmatrix} z(t) \rightsquigarrow \dot{x} = Ax + Bu$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} z(t) \rightsquigarrow y = Cx + Du$$

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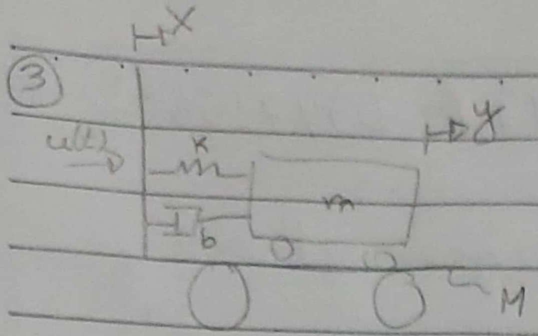

$$\begin{cases} M\ddot{x} - b(\dot{y} - \dot{x}) - K(y - x) = u \\ m\ddot{y} + b(\dot{y} - \dot{x}) + K(y - x) = 0 \end{cases}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K/M & +K/M & -b/M & +b/M \\ K/M & -K/M & b/M & -b/M \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/M \\ 0 \end{bmatrix} u \rightarrow \dot{x} = Ax + Bu$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & & 0 & 0 \\ & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u \rightarrow y = Cx + Du$$

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(4) 4- Sendo $\dot{x} = \begin{bmatrix} \dot{x}_G & \dot{\theta} & \dot{x}_1 & \dot{x}_2 & \ddot{x}_G & \ddot{\theta} & \ddot{x}_1 & \ddot{x}_2 \end{bmatrix}$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{(K_1+K_2)}{M} & -\frac{l}{2M}(K_1-K_2) & +\frac{K_1}{M} & +\frac{K_2}{M} & -\frac{(b_1+b_2)}{M} & -\frac{l}{2}\frac{(b_1-b_2)}{M} & +\frac{b_1}{M} & +\frac{b_2}{M} \\ -\frac{(K_1-K_2)l}{2J} & -\frac{l^2}{4}\frac{(K_1+K_2)}{J} & \frac{K_1 l}{2J} & -\frac{K_2 l}{2J} & -\frac{(b_1-b_2)l}{2J} & -\frac{l^2}{4}\frac{(b_1+b_2)}{J} & \frac{b_1 l}{2J} & -\frac{b_2 l}{2J} \\ \frac{K_1}{m_1} & \frac{l K_1}{2m_1} & -\frac{2K_1}{m_1} & 0 & \frac{b_1}{m_1} & \frac{l b_1}{2m_1} & -\frac{b_1}{m_1} & 0 \\ \frac{K_2}{m_2} & -\frac{l}{2}\frac{K_2}{m_2} & 0 & -\frac{2K_2}{m_2} & \frac{b_2}{m_2} & -\frac{l}{2}\frac{b_2}{m_2} & 0 & -\frac{b_2}{m_2} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ K_1/m_1 & 0 \\ 0 & K_2/m_2 \end{bmatrix} \qquad u = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ z(t) \\ z(t+\alpha) \end{bmatrix}$$

$$\dot{x} = Ax + Bu$$

⑤ $$\begin{cases} (M+m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2 \operatorname{sen}\theta = u \\ ml\cos\theta\,\ddot{x} + J\ddot{\theta} = mgl\operatorname{sen}\theta \end{cases}$$

⇓ linearizando em torno de $\theta = 0$

$$\begin{cases} (M+m)\ddot{x} + ml\ddot{\theta} = u & \text{(I)} \\ ml\ddot{x} + J\ddot{\theta} = mg\theta l & \text{(II)} \end{cases}$$

↳ Por (I): $\ddot{x} = \dfrac{u - ml\ddot{\theta}}{M+m}$

↳ Substituindo em (II)

$$ml\left(\frac{u - ml\ddot{\theta}}{M+m}\right) + J\ddot{\theta} = mg\theta l$$

$$\ddot{\theta}\left(J - \frac{m^2 l^2}{M+m}\right) = mg\theta l - \frac{mlu}{M+m}$$

$$\boxed{\ddot{\theta} = \left[mg\theta l - \frac{mlu}{M+m}\right] \Big/ \left(J - \frac{m^2 l^2}{M+m}\right)}$$

↳ Substituindo em (II)

$$ml\ddot{x} + J\ddot{\theta} = mg\theta l$$

$$\ddot{x} = \frac{mg\theta l - J\ddot{\theta}}{ml} = \boxed{\frac{mg\theta l}{ml} - \frac{J}{ml}\,\frac{\left[mg\theta l - \frac{mlu}{M+m}\right]}{J - \frac{m^2 l^2}{M+m}}}$$

⑥ $\begin{cases} m\ddot{x} = mg - Ki^2/x^2 \\ L\,di/dt + Ri = V(t) \end{cases}$

↳ Linearizando

$$\ddot{x} - \ddot{x}_{eq} = \frac{2Ki_{eq}^2}{m x_{eq}^3} \cdot (x - x_{eq}) - \frac{2Ki_{eq}}{m x_{eq}^2} (i - i_{eq})$$

$$\delta\ddot{x} = \frac{2Ki_{eq}^2}{m x_{eq}^3}\,\delta x - \frac{2Ki_{eq}}{m x_{eq}^2}\,\delta i$$

$$\begin{bmatrix} \delta\dot{x} \\ \delta\ddot{x} \\ \delta\dot{i} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2Ki_{eq}^2}{m x_{eq}^3} & 0 & -\frac{2Ki_{eq}}{m x_{eq}^2} \\ 0 & 0 & -R/L \end{bmatrix} \begin{bmatrix} \delta x \\ \delta\dot{x} \\ \delta i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix} \delta V(t)$$

$\sim \dot{x} = Ax + Bu$

$$\begin{bmatrix} \delta x \\ \delta\dot{x} \\ \delta i \end{bmatrix} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \begin{bmatrix} \delta x \\ \delta\dot{x} \\ \delta i \end{bmatrix} \rightsquigarrow y = Cx + Du$$